D 100611 (Pages : 3) Name.................................

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SIXTH SEMESTER UG (CBCSS-UG) DEGREE EXAMINATION, MARCH 2024

Mathematics

MTS 6B 10—REAL ANALYSIS

(2019 Admissions onwards)

Time : Two Hours and a Half

Maximum Marks : 80

Section A

Questions 1—15. *Answer any number of questions. Each carry* 2 *marks. Maximum marks* 20.

1. State discontinuity criterion. Hence show that the signum function is not continuous at $x = 0$.
2. State maximum-minimum theroem.
3. Show that $f(x) = \frac{1}{x}$ is uniformly continuous on $[a, \infty)$ where $a > 0$.
4. Define Riemann integral of a function f on an integral $[a, b]$.
5. If f and g are in $R[a, b]$ and if $f(x) \leq g(x)$ for all x in $[a, b]$ then show that $\int_a^b f \leq \int_a^b g$.
6. State Lebesgue's integrability criterion.
7. If f and g belong to $R[a, b]$ then the product fg belongs to $R\,[a, b]$.
8. Show that $\lim \frac{\sin(nx+n)}{n} = 0$ for $x \in \mathbb{R}$.
9. Discuss the uniform convergence of $f_n(x) = \frac{x}{n}$ on $A = [0, 1]$.
10. Evaluate $\lim\left(e^{-nx}\right)$ for $x \in \mathbb{R}, x \geq 0$.
11. Define absolute convergence of series of functions.
12. Evaluate $\int_{-\infty}^{0} e^x \, dx$.
13. Find the principal value of $\int_{-\infty}^{\infty} \frac{dx}{x^2+1}$.

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14. Show that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

15. Define Beta fucntion. St B (p, q) = B (q, p).

Section B

Questions 16—23. Answer any number of questions.
Each carry 5 marks. Maximum marks 35.

16. Let $A=\{x \in \mathbb{R} | x>0\}$. Define h on A by $h(x)=0$ if $x \in A$ is irrational and $h(x)=\frac{1}{n}$ if x rational with $x=\frac{m}{n}, m, n \in \mathbb{N}$ have no common factor except 1. Then show that is continuous at every irrational number in A and discontinuous at every rational number in A.

17. Let I be an interval and $f: I \rightarrow \mathbb{R}$ be a continuous function on I then show that $f(I)$ interval.

18. If $f \in R\,[a, b]$ then show that f is bounded on $[a, b]$.

19. Show that if $\phi:[a, b] \rightarrow R$ is a step function then $\phi \in R[a, b]$.

20. Evaluate $\lim \frac{x^2+nx}{n}, x \in R$. Is the convergence uniform on $\mathbb{R}$?

21. Let (f_n) be a sequence of bounded functions on $A \subseteq \mathbb{R}$. Then show that (f_n) converges uniformly on A to a bounded function f iff for each $\varepsilon>0$ there is a number $H(\varepsilon)$ in $\mathbb{N}$ that for all $m, n \geq H(\varepsilon)$ then $\|f_m-f_n\|_A \leq \varepsilon$.

22. Discuss the convergence of $\int_0^\infty \frac{\sin^2 x}{x^2} dx$.

23. Define Beta function and show that $\forall p>0, q>0, B(p, q)=2 \int_0^{\pi/2} \sin^{2p-1} \theta \cos^{2p-1} \theta \, d\theta$.

Section C

Questions 24—27. Answer any **two** *questions.*
Each carry 10 marks.

24. (a) Show that if f and g are uniformly continuous on $A \subseteq \mathbb{R}$ and they are bounded then their product fg is also uniformly continuous.

    (b) Show that $f(x)=\sqrt{x}$ is uniformly continuous on $[a, \infty)$ where $a>0$.

25. Suppose f and g are in R $[a, b]$. Then

(a) if $k \in R$, show that $kf \in \mathcal{R}[a,b]$ and $\int_a^b kf = k\int_a^b f$.

(b) $f + g \in \mathcal{R}[a,b]$ and $\int_a^b f + g = \int_a^b f + \int_a^b g$.

26. Discuss the pointwise and uniform convergence of :

(a) $f_n(x) = \dfrac{\sin(nx+n)}{n}$ for $x \in \mathbb{R}$.

(b) $g_n(x) = \dfrac{x^2 + nx}{n}$ for $x \in \mathbb{R}$.

27. Show that $\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}$.

(2 × 10 = 20 marks)