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Name.....

Reg. No.....

**SIXTH SEMESTER UG (CBCSS-UG) DEGREE
EXAMINATION, MARCH 2024**

Mathematics

MTS 6B 11—COMPLEX ANALYSIS

(2020 Admissions onwards)

Time : Two Hours and a Half

Maximum Marks : 80

Section A*Answer any number of questions.**Each question carries 2 marks. Maximum marks 25.*

1. Show that $f(z) = \bar{z}$ is nowhere differentiable.
2. Verify Cauchy-Riemann equations for $f(z) = z^2$.
3. Write the Cauchy-Riemann equations in polar co-ordinates.
4. Define harmonic function and harmonic conjugate function.
5. Solve $e^w = -2$.
6. Express $\cos(2 - 4i)$ in the form $a + ib$.
7. Evaluate $\int y dx + x dy$ on the curve $y = x^2$ from $(0, 0)$ to $(1, 1)$.
8. Evaluate $\oint z dz$ over the first quadrant of the circle $|z| = 1$ from $z = i$ to $z = 1$.
9. Prove that $\oint_C f(z) dz = 0$ for $f(z) = \frac{z^2}{z-3}$ where C is the unit circle $|z| = 1$.
10. Evaluate $\oint_C \frac{z+1}{z^4 + 2iz^3} dz$ where C is the circle $|z| = 1$.
11. Define (a) Power series ; (b) Circle of convergence.
12. Write the Maclaurin series expansion for $\sin z$ and $\cos z$.
13. Expand $f(z) = e^{\frac{3}{z}}$ in a Laurent series valid for $0 < |z| < \infty$.
14. Find zeroes of $f(z) = \frac{z-2}{z^2} \sin \frac{1}{z-1}$.
15. Find the residue of $f(z) = \frac{z}{z^2 + 1}$ at its poles.

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Section B

Answer any number of questions.

Each question carries 5 marks. Maximum marks 35.

16. Verify the Cauchy Riemann equations for $f(x+iy) = \frac{x-iy}{x^2+y^2}$.
17. If $f(z) = u+iv$ then show that $|f'(z)|^2 = u_x^2 + v_x^2 = u_y^2 + v_y^2$.
18. Find the image of the annulus $2 \leq |z| \leq 4$ under the mapping $w = \text{Ln } z$.
19. Find the principal value of $(-3)^{\frac{i}{\pi}}$.
20. Using Cauchy-Goursat theorem, evaluate $\oint_C \frac{5z+7}{z^2+2z-3} dz$ where C is the circle
21. Find $\oint_C \frac{z^2-4z+4}{z+i} dz$ where C is the circle $|z| = 2$.
22. Prove that the sequence $\left\{ \frac{3+ni}{n+2ni} \right\}$ converges to $\frac{2}{5} + \frac{1}{5}i$.
23. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

Section C

Answer any two questions.

Each question carries 10 marks. Maximum marks 20.

24. Verify that the function $u(x,y) = x^3 - 3xy^2 - 5y$ is harmonic in the entire complex plane. Find the harmonic conjugate function.
25. State and prove (a) Liouville's theorem ; (b) Morera's theorem.
26. State Cauchy's residue theorem, and using this show that $\oint_C \frac{dz}{z \sin z} = 0$, where C is a circle about the origin described in the positive sense.
27. Show that $\int_0^{2\pi} \frac{d\theta}{2 + \cos \theta} = \frac{2\pi}{\sqrt{3}}$.