

(Pages : 4)

Name.....

Reg. No.....

## FIFTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTS 5D 03—LINEAR MATHEMATICAL MODELS

(2020 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

## Section A

Answer any number of questions.  
Each question carries 2 marks.  
Ceiling is 20.

1. Find the equation of the line through  $(0, -3)$  with slope  $\frac{3}{4}$ .
2. Find  $k$  such that the line through  $(4, -1)$  and  $(k, 2)$  is parallel to  $2x + 3y = 6$ .
3. Solve the system of equations :

$$2x + 3y - z = 1$$

$$3x + 5y + z = 3$$

where  $z$  is the parameter.

$$4. \text{ Let } A = \begin{bmatrix} -2 & 4 \\ 0 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} -6 & 2 \\ 4 & 0 \end{bmatrix}. \text{ Find } -4A + 5B.$$

$$5. \text{ Find the inverse of } B = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}.$$

6. Graph the linear inequality  $y \geq x - 3$ .
7. Explain briefly the steps to use in solving a linear programming problem by graphical method.
8. Graph the solution of each of the following inequalities and tell whether the region is bounded or unbounded :

$$x + y \leq 6$$

$$2x - y \geq 3.$$

Turn over

9. Explain standard maximum form of a linear programming problem.
10. Introduce slack variables as necessary and write the initial simplex tableau for the linear programming problem given below :

Find  $x_1 \geq 0$  and  $x_2 \geq 0$  such that

$$4x_1 + 2x_2 \leq 5$$

$$x_1 + 2x_2 \leq 4$$

and  $z = 7x_1 + x_2$  is maximized.

11. Find the transpose of the matrix  $A = \begin{bmatrix} 4 & 5 & -3 & 15 \\ 7 & 14 & 20 & -8 \\ 5 & 0 & -2 & 23 \end{bmatrix}$ .

12. State the dual of the linear programming problem given below :

$$\text{Maximize } z = 4x_1 + 3x_2 + 2x_3$$

subject to  $x_1 + x_2 + x_3 \leq 5$

$$x_1 + x_2 \leq 4$$

$$2x_1 + x_2 + 3x_3 \leq 15$$

with  $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$ .

### Section B

Answer any number of questions.

Each question carries 5 marks. Ceiling is 30.

13. A firm producing poultry feed finds that the total cost  $C(x)$  in dollars of producing and units is given by  $C(x) = 20x + 100$ . Management plans to charge \$24 per unit for the feed.

(a) How many units must be sold for the firm to break-even?

(b) What is the profit if 100 units of feed are sold?

(c) How many units must be sold to produce a profit of \$900?

14. Use Gauss-Jordan method to solve the following system of equations :

$$x + 5z = -6 + y$$

$$3x + 3y = 10 + z$$

$$x + 3y + 2z = 5$$

15. Find  $A^{-1}$  if  $A = \begin{bmatrix} 1 & 0 & 1 \\ 2 & -2 & -1 \\ 3 & 0 & 0 \end{bmatrix}$ .

16. Find the maximum value of the objective function  $Z = 3x + 4y$  subject to the following constraints

$$2x + y \leq 4$$

$$-x + 2y \leq 4$$

$$x \geq 0, y \geq 0$$

17. Set up the initial simplex tableau for the following problem :

A farmer has 100 acres of available land on which he wishes to plant a mixture of potatoes, corn, and cabbage. It costs him \$400 to produce an acre of potatoes, \$160 to produce an acre of corn, and \$280 to produce an acre of cabbage. He has a maximum of \$20,000 to spend. He makes a profit of \$120 per acre of potatoes, \$40 per acre of corn, and \$60 per acre of cabbage. How many acres of each crop should he plant to maximize his profit?

18. Pivot about the indicated 2 of the following initial simplex tableau :

$x_1$	$x_2$	$x_3$	$s_1$	$s_2$	$s_3$	$z$	
<u>2</u>	1	1	1	0	0	0	150
1	2	8	0	1	0	0	200
2	3	1	0	0	1	0	320
-3	-2	-1	0	0	0	1	0

19. Explain briefly the steps involved in solving a standard maximum linear programming problem by simplex method.

### Section C

Answer any **one** question.  
The question carries 10 marks.

20. Solve the following system of equations by using inverse of the coefficient matrix :

$$-x - y - z = 1$$

$$4x + 5y = -2$$

$$y - 3z = 3$$

Turn over

21. Use simplex method to solve the following linear programming problem :

$$\text{Maximize } z = 12x_1 + 10x_2$$

$$\text{subject to } x_1 + 2x_2 \geq 24$$

$$x_1 + x_2 \leq 40$$

$$x_1 \geq 0, x_2 \geq 0.$$

(1 × 10 = 10 marks)