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Name.....

Reg. No.....

FIFTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION NOVEMBER 2022

Mathematics

MTS 5B 06—BASIC ANALYSIS

(2020 Admission onwards)

Time: Two Hours and a Half

Maximum : 80 Marks

Section A

Answer any number of questions. Each question carries 2 Marks. Maximum 25 Marks.

- 1. Define denumerable set. Give an example.
- 2. If $a \in \mathbb{R}$, then prove that $a \cdot 0 = 0$.
- 3. Let a, b, c be elements of \mathbb{R} and if a > b and b > c, then prove that a > c.
- 4. Prove that |-a| = |a| for all $a \in \mathbb{R}$.
- 5. Describe Fibonacci sequence.
- 6. State Monotone Convergence Theorem.
- 7. Define Cauchy sequence. Give an example.
- 8. Define properly divergent sequence.
- 9. Show that $\mathbb{R} = (-\infty, \infty)$ is open.
- 10. Describe any two properties of Cantor Set.
- 11. Whether the sequence $\left(1, \frac{1}{2}, 3, \frac{1}{4}, \dots\right)$ is convergent? Justify your answer.
- 12. Find the principal cube root at the point z = i.
- 13. Define bounded subset of the complex plane.

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- 14. Find the reciprocal of z = 2 3i.
- 15. Express $-\sqrt{3} i$ in polar form.

Section B

Answer any number of questions. Each question carries 5 Marks. Maximum 35 marks.

- 16. State and prove Cantor's Theorem.
- 17. Prove that there does not exist a rational number r such that $r^2 = 2$.
- 18. (a) Define supremum of a set of real numbers.
 - (b) Prove that there can be only one supremum of a given subset S of R, if it exists.
- 19. Prove that $\lim \left(\frac{1}{n^2}\right) = 0$.
- 20. If 0 < b < 1, then prove that $\lim_{n \to \infty} (b^n) = 0$.
- Prove that the intersection of an arbitrary collection of closed sets in $\mathbb R$ is closed.
- Show that the complex function f(z) = z + 3i is one-to-one on the entire complex plane and f(z) = z + 3iformula for its inverse function.
- 23. If $f(z) = \frac{z}{\overline{z}}$ then show by two path test that $\lim_{z \to 0} f(z)$ does not exist.

Section C

Answer any two questions. Each question carries 10 marks.

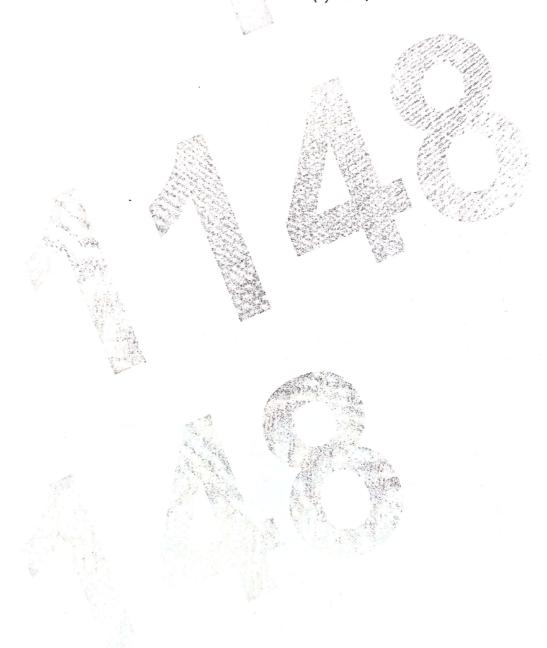
- 24. State and prove Monotone Subsequence Theorem.
- 25. Prove that a monotone sequence of real numbers is convergent if and only if it is bound $X = (x_n)$ is a bounded increasing sequence, then prove that :

$$\lim (x_n) = \sup \{x_n : n \in \mathbb{N}\}.$$

- 26. Let $X=(x_n)$ be a sequence of real numbers that converges to x and suppose that $x_n \ge 0$. Then prove that the sequence $(\sqrt{x_n})$ converges and $\lim (\sqrt{x_n}) = \sqrt{x}$.
- 27. Show that the function f defined by:

$$f(z) = \sqrt{r}e^{i\theta/2}, -\pi < \theta < \pi$$

is a branch of the multiple-valued function $F(z) = z^{1/2}$.



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