

FIFTH SEMESTER (CBCSS-UG) DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTS 5B 09—INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS
(2020 Admission onwards)

Time : Two Hours

Maximum : 60 Marks

Section A*Answer any number of questions.**Each question carries 2 marks.**Ceiling is 20.*

1. Find equation of the normal to the parabola $y^2 = x$ at the point (1,1).
 2. Find the foci of the hyperbola $\frac{x^2}{9} - \frac{y^2}{4} = 1$.
 3. What is the reflection property of the hyperbola?
 4. Find the matrix form of the conic $11x^2 + 4xy + 14y^2 - 4x - 28y - 16 = 0$.
 5. Show that $x^3 + x^2 - 5x + 3$ is divisible by $x + 3$.
 6. Write a cubic equation with the roots 1,2,3.
 7. State the Identity Theorem.
 8. Find the multiplicity of the root $x = 1$ of the polynomial $f(x) = x^n - nx + n - 1$.
 9. Show that the polynomial $f(x) = x^{11} - 1$ has no roots in the interval $(-1,0)$.
 10. Find Δ of the equation $x^3 - 10x - 12 = 0$.
 11. Show that $\sqrt{2} - \sqrt{3}$ is a root of the equation $x^4 - 10x + 1 = 0$.
 12. Find the cubic resolvent corresponding to the bi quadratic equation $x^4 + 4x - 1 = 0$.
- (Ceiling 20)

Turn over

Section B

Answer any number of questions.
Each question carries 5 marks.
Ceiling is 30.

13. Show that $t(x) = \begin{pmatrix} 1 & 3 \\ 1 & 2 \end{pmatrix}x + \begin{pmatrix} 4 \\ -2 \end{pmatrix}$ is an affine transformation and find the inverse.
14. (i) State the Fundamental theorem of Affine Geometry.
(ii) Determine the affine transformation which maps the points $(0, 0)$, $(1, 0)$ and $(0, 1)$ to points $(3, 2)$, $(5, 8)$ and $(7, 3)$, respectively.
15. Find the rational roots of the equation $6x^4 - 7x^3 + 8x^2 - 7x + 2 = 0$.
16. Solve the equation $3x^3 - 16x^2 + 23x - 6 = 0$ if the product of two roots is 1.
17. Factorize in to real linear and quadratic factors of the polynomial $f(x) = x^4 + 1$.
18. Show that the necessary and sufficient condition for an equation $x^3 + px + 1 = 0$ to have three distinct roots is $p^3 < -27/4$.
19. How many real roots of the equation $f(x) = x^4 - 32x + 1 = 0$.

Section C

Answer any **one** question.
The question carries 10 marks.

20. Prove that the conic with the equation $3x^2 - 10xy + 3y^2 + 14x - 2y + 3 = 0$ is a hyperbola, find its centre, and its major and minor axis.
21. Solve the cubic equation $x^3 + x^2 - 2 = 0$ by using Carden's formula.

(1 × 10 = 10)