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Name.....

Reg. No.....

**FIFTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION
NOVEMBER 2022**

Mathematics

MTS 5B 05—THEORY OF EQUATIONS AND ABSTRACT ALGEBRA

(2019 Admissions only)

Time : Two Hours and a Half

Maximum : 80 Marks

Section A

Answer any number of questions.

Each question carries 2 marks.

Ceiling is 25.

1. Find the quotient and remainder when $-x^4 + 7x^3 - 4x^2$ is divided by $x - 3$.
2. Write a cubic equation with roots $1, 1 + i, 1 - i$.
3. Factorise into real linear and quadratic factors : $x^4 + x^3 + x^2 + x + 1$.
4. State Rolles Theorem.
5. Verify whether the equation $x^3 - 3x^2 - 4x + 13 = 0$ has roots in the interval $(-3, -2)$.
6. Define Euler's function. Find the value of $\phi(36)$.
7. Find the multiplicative inverse of (14) in \mathbb{Z}_{15} .
8. Prove that if p is a prime $(p-1)! \equiv -1 \pmod{p}$.
9. Check whether the relation \sim on \mathbb{R} defined by $a \sim b$ if $|a - b| \leq 1$ is an equivalence relation.
10. Let $\sigma, \tau \in S_7$ be given by $\sigma = (1\ 3\ 5\ 6)$ and $\tau = (1\ 2)(3\ 5\ 4\ 7)$. Find $\sigma \tau \sigma^{-1}$.
11. Find the order of $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 8 & 4 & 9 & 6 & 5 & 2 & 3 & 1 & 7 \end{pmatrix}$.

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12. Let G be a group. Prove that if $x^2 = e$ for all $x \in G$ then G is abelian.
13. Write the subgroups of S_3 .
14. Let G be a group with $a, b \in G$ then show that $O(aba^{-1}) = O(b)$.
15. Let $G = \mathbb{Z}_{12}$ and $H = 4\mathbb{Z}_{12}$. Find all cosets of H .

Section B

Answer any number of questions.

Each question carries 5 marks.

Ceiling is 35.

16. Solve the biquadratic equation $x^4 - 7x^3 + 18x^2 - 22x + 12 = 0$ whose roots are a, b, c and d such that $ab = 6$.
17. Find the limits of roots of the equation $2x^5 - 8x^4 - 11x^3 + 5x^2 + 2x - 11 = 0$.
18. Separate the roots of the equation $3x^4 - 2x^3 - 6x^2 + 6x - 2 = 0$.
19. Let G be a group and $a, b \in G$. Prove that $(ab)^n = a^n b^n$ for all $n \in \mathbb{Z}$ iff $ab = ba$.
20. Find all cyclic subgroups of \mathbb{Z}_8 .
21. Check whether the set $\{a + b\sqrt[3]{2} \mid a, b \in \mathbb{Q}\}$ are subring of the field of real numbers.
22. Show that the cyclic group $\langle i \rangle$ is isomorphic to \mathbb{Z}_4 .
23. Let $\phi: G \rightarrow H$ is an isomorphism of groups. Then show that :
- a) If G is abelian then so is H ; and
 - b) If G is cyclic then so is H .

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Section C

Answer any **two** questions.
Each question carries 10 marks.
Maximum marks 20.

24. Solve the cubic equation : $2x^3 + 3x^2 + 3x + 1 = 0$.
25. Let G be a group with normal subgroups H and K such that $HK = G$ and $H \cap K = \{e\}$ then show that $G \cong H \times K$.
26. Let G be a cyclic group. Then show that :
- a) If G is infinite then $G \cong \mathbb{Z}$.
 - b) If G is finite with $|G| = n$, then $G \cong \mathbb{Z}_n$.
27. State and prove Fundamental Homomorphism theorem.

(2 × 10 = 20 marks)