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C 3555 (Pages : 4) Name..

Reg. No......................................

FOURTH SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MTS 4B 04—LINEAR ALGEBRA

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Type Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Describe different possibilities for solution (x, y) of a system linear equations in the xy plane. What are consistent system ?

2. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$ solve the system.

3. Define trace of a square matric. Find the trace of the matrix $A = \begin{bmatrix} -1 & 2 & 7 & 0 \\ 3 & 5 & -8 & 4 \\ 1 & 2 & 7 & -3 \\ 4 & -2 & 1 & 0 \end{bmatrix}$.

4. Show that the standard unit vectors

   $e_1 = (1, 0, ...0), e_2 = (0, 1, 0....0), e_3 = (0, 0, 1, 0....0)......e_n = (0, 0......1)$ span R^n.

5. Find the co-ordinate vector of $w = (1, 0)$ relative to the basis $s = [\bar{u}_1, \bar{u}_2]$ of R^2, where $\bar{u}_1 = (1, -1)$ and $\bar{u}_2 = (1, 1)$.

6. Write two important facts about the vectors in a finite dimensional vector space V.

Turn over

7. Consider the bases $B = [\bar{u}_1, \bar{u}_2]$ and $B' = [\bar{u}_1', \bar{u}_2']$ where

$\bar{u}_1 = (1, 0), \bar{u}_2 = (0, 1), \bar{u}_1' = (1, 1), \bar{u}_1' = (2, 1)$. Find the transition matrix $P_{B' \to B}$ from B' to B

8. Define row spaces and null spaces an $m \times n$ matrix.

9. If $R = \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{3}{2} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ is the row reduced echelon form of a 3×3 matrix A, then verify the

nullity formula.

10. Show that the operator $T : R^2 \to R^2$ that rotates vectors through an angle θ is one-one.

11. Find the image of the line $y = 4x$ under multiplication by the matrix $A = \begin{bmatrix} 5 & 2 \\ 2 & 1 \end{bmatrix}$.

12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen val

$A = \begin{bmatrix} 3 & 0 \\ 8 & -1 \end{bmatrix}$ $x = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

13. Let A be an $n \times n$ matrix. Define inner product on R^n generated by A. Also write the genera matrix of the weighted Euclidear inner product $< u, v > = w_1u_1v_1 + w_2u_2v_2 + ... + w_nu_nv_n$.

14. If u, v are vectors in a real inner product space V, then show that $\| u + v \| \leq \| u \| + \| v \|$.

15. If A is an $n \times n$ orthogonal matrix, then show that $\| Ax \| = \| x \|$ for all x in R^n.

(10 × 3 = 30 ma

Section B (Paragraph/Problem Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

6. Describe Column Row Expansion method for finding the product AB for two matrices A and B. Use this to find the product $AB = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix} \begin{bmatrix} 2 & 0 & 4 \\ -3 & 5 & 1 \end{bmatrix}$.

7. If A is an invertible matrix, then show that A^T is also invertible and $(AT)^{-1} = \left(A^{-1}\right)^T$.

8. Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in R^3. Show that $w = (9, 2, 7)$ is a linear combination of u and v and that $w' = (4, -1, 8)$ is not a linear combination of u and v.

9. If $s = \{v_1, v_2 ....v_n\}$ is a basis for a vector space V, then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + ... + c_nv_n$ in exactly one way. What are the co-ordinates of v relative to the basis s.

0. If A is a matrix with n columns, then define rank of A and show that rank (A) + nullity $(A) = n$.

1. Find the standard matrix for the operator $T : R^3 \to R^3$ that first rotates a vector counter clockwise about z-axis through an angle θ, then reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.

2. Define eigen space corresponding to an eigen value λ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

3. If w is a sub-space of real inner product space v, then show that :

   (a) $w^{\perp}$ is subspace of v.

   (b) $w \cap w^{\perp} = \{0\}$.

(5 × 6 = 30 marks)

Turn over

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. (a) Show that every elementary matrix is invertible and the inverse is also an elementary

(b) Find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 3 \\ 1 & 0 & 8 \end{bmatrix}$ using Row operations.

25. (a) Let V be a vector space and $\bar{u}$ a vector in V and K a scalar. Then show that :

(a) $0\,\bar{u} = 0$; and

(b) $(-1)\,\bar{u} = -\bar{u}$.

(b) Show that the vectors $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$ form a basis for R^3.

26. (a) Consider the basis $B = [u_1, u_2]$ and $B' = [u_1{}^1, u_2{}^1]$ for R^2 where $u_1 = (2, 2)$, $u_2 =$

$u_1{}' = (1, 3)$, $u_2{}' = (-1, -1)$

(i) Find the transition matrix B′ to B.

(ii) Find the transition matrix B to B′.

(b) Find the reflection of the vector $x = (1, 5)$ about the line through the origin that ma

angle of $\frac{\pi}{6}$ with the x-axis.

27. When you can say that a square matrix A is diagonalizable ? If A is an $n \times n$ matrix, show th following statements are equivalent :

(a) A is diagonalizable ; and

(b) A has n linearly independent eigen vectors.

$(2 \times 10 = 20$ n