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Name.......................................

Reg. No.....................................

SIXTH SEMESTER U.G. DEGREE EXAMINATION, MARCH 2022

(CBCSS–UG)

Mathematics

MTS 6B 12—CALCULUS OF MULTIVARIABLE

(2019 Admissions)

me : Two Hours and a Half

Maximum : 80 Marks

Section A (Short Answer Questions)

Answer at least **ten** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 30.

1. Find the domain and rang of the function $f(x,y)=x+3y-1$.

2. Evaluate $\lim_{(x,y,z)\to\left(\frac{\pi}{2},0,1\right)} \frac{e^{2y}(\sin x+\cos y)}{1+y^2+z^2}$.

3. Find f_x and f_y if $f(x,y)=x\cos xy^2$.

4. Find the directional derivative of $f(x,y)=x^2\sin 2y$ at $\left(1,\frac{\pi}{2}\right)$ in the direction of $\bar{v}=3\hat{i}-4\hat{j}$.

5. Find $\nabla f(x,y,z)$ if $f(x,y,z)=x^2+y^2-4z$ and find the direction of maximum increase of f at the point (2, –1, 1).

6. Find the relative extrema of the function $f(x,y)=x^2+y^2-2x+4y$.

7. Find the volume of the solid lying under the elliptic paraboloid $z=8-2x^2-y^2$ above the rectangular region given by $0\le x\le 1, 0\le y\le 2$.

8. Find the mass of the triangular lamina with vertices $(0,0),(0,3)$ and $(2,3)$ given that the density at (x, y) is $\rho(x,y)=2x+y$.

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9. Find the Jacobian for the change of variables defined by $x = r\cos\theta, y = r\sin\theta$.

10. Evaluate $\iint_R 2x - y\, dA$ where R is the region bounded by the parabola $x = y^2$ and the line $x - y$

11. Find whether the vector field $\vec{F} = x^2 y\,\hat{i} + xy\,\hat{j}$ is conservative.

12. State Green's theorem.

13. Find a parametric representation for the cone $x = \sqrt{y^2 + z^2}$.

14. Find the surface area of the torus given by $\vec{r}(u,v) = (2+\cos u)\cos v\,\hat{i} + (2+\cos u)\sin v\,\hat{j} + \sin$ where the Domain D is given by $0 \le u \le 2\pi$ and $0 \le v \le 2\pi$.

15. Compute $\iint_S \vec{F}\cdot\hat{n}\, dS$ given $\vec{F}(x,y,z) = (x+\sin z)\,\hat{i} + (2y+\cos x)\,\hat{j} + (3z+\tan y)\,\hat{k}$ and S is the sphere $x^2 + y^2 + z^2 = 1$.

(10 × 3 = 30 ma

Section B (Paragraph Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Find f_{xyx} and f_{yx_y} if $f(x,y) = x\cos y + y\sin x$.

17. Find the differential of $w = x^2 + xy + z^2$. Compute the value of dw if (x, y, z) changes from (1 to $(0.98, 2.03, 1.01)$ and compare the value with that of Δw.

18. Find the equation of the tangent plane and normal line to the surface $x^2 - 2y^2 - 4$ at (4, –2, –1).

19. Find the relative extrema of $f(x,y) = -x^3 + 4xy - 2y^2 + 1$.

20. Find the volume of the solid region bounded by the paraboloid $z = 4 - x^2 - 2y^2$ and the xy

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21. Evaluate $\int_0^2 \int_0^x \int_0^{x+y} e^x (y+2z)\, dz\, dy\, dx$.

22. Determine whether the vector field $\vec{F} = e^x \left(\cos y\, \hat{i} - \sin y\, \hat{j}\right)$ is conservative. If so, find a potential function for the vector field.

23. Evaluate $\oint_C \left(e^x + y^2\right) dx + \left(x^2 + 3xy\right) dy$, where C is the positively oriented closed curve lying on the boundary of the semi annular region R bounded by the upper semicircles $x^2 + y^2 = 1$ and $x^2 + y^2 = 9$ and the x-axis.

(5 × 6 = 30 marks)

Section C (Essay Questions)

Answer any **two** *questions.*
Each question carries 10 marks.

24. (a) Sketch the graph of $f(x,y) = 9 - x^2 - y^2$.

 (b) Show that $\lim_{(x,y)\to(0,0)} \frac{xy}{x^2+y^2}$ does not exist.

25. (a) Find the relative extrema of $f(x,y) = x^3 + y^2 - 2xy + 7x - 8y + 2$.

 (b) Find the minimum value of $f(x,y,z) = 2x^2 + y^2 + 3z^2$ subject to the constraint $2x - 3y - 4z = 49$.

26. Let R be the region bounded by the square with vertices $(0,1), (1,2), (2,1)$ and $(1,0)$. Evaluate $\iint_R (x+y)^2 \sin^2(x-y)\, dA$.

27. Let $\vec{F} = (x,y,z) = 2xyz^2\, \hat{i} + x^2 z^2\, \hat{j} + 2x^2 yz\, \hat{k}$.

 (a) Show that $\vec{F}$ is conservative and find a scalar function f such that $\vec{F} = \nabla f$.

 (b) If $\vec{F}$ is a force field, find the work done by $\vec{F}$ in moving a particle along any path from (0, 1, 0) to (1, 2, −1).

(2 × 10 = 20 marks)

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