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1250 (Pages : 3) Name.....................................

Reg. No.................................

SIXTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION MARCH 2021

Mathematics

MAT 6B 13 (E01)—GRAPH THEORY

ne : Three Hours Maximum : 80 Marks

Section A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Fill in blanks : The number of vertices in a complete bipartite graph $K_{4,3}$ is ————.
2. Define Self complementary graph.
3. Define the neighborhood set of a vertex v in a graph G.
4. Define incidence matrix of a graph G.
5. Fill in blanks : Let G be a graph with n vertices $v_1, v_2, \ldots\ldots v_n$ and let A denote the adjacency matrix of G. Let k be any positive integer. Then the $(i, j)^{th}$ entry of A^k is ————.
6. Define bridge of a graph G.
7. How many different spanning trees for a complete graph K_4.
8. Give an example for Euler graph.
9. Define Tree.
0. Define the eccentricity of a vertex v in a graph G.
1. Draw Peterson graph.
2. Define the vertex connectivity of a graph.

$(12 \times 1 = 12$ marks)

Section B

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

3. State the First theorem of Graph Theory.

1. For a graph with n–vertices and m edges, prove that $\delta(G) \leq \dfrac{2m}{n} \leq \Delta(G)$.

Turn over

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15. Prove that for any *two* vertices u and v of a graph G, every $u - v$ walk contains a $u - v$ path.
2.

16. Define k-regular graph. Give an example for 3-regular graph.
3.

17. Define edge deleted sub graph of a graph G with example.

18. Give an example for Wheel graph.

19. Prove that the complete bipartite graph $K_{m,n}$ is the join of the complements of K_m and K_n.

20. How many vertices and edges for the k-cube graph Q_k ?

21. Prove that if T is a spanning tree of G which contains e then $T - e$ is a spanning tree of $G - e$.
4.

22. State Whitney theorem.

23. Define Hamiltonian graph.
5.

24. State Euler's formula.

(8 × 3 = 24 mark

Section C

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.
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25. Prove that for any simple graph G, there is an even number of odd degree vertices.

26. Let G be an acyclic graph with n vertices and k connected components. Then prove that G h $n - k$ edges.

27. Let G be a graph with n vertices $v_1, v_2, \ldots\ldots v_n$. Let A be the adjacency matrix of G with respect this listing of the vertices. Let $B = (b_{ij})$ be the matrix defined by $B = A + A^2 + \ldots\ldots\ldots\ldots A^{n-}$ Prove that G is a connected graph if and only if $b_{i,j} \neq 0, \forall i \neq j$.

28. Prove that a graph G is connected if and only if it has a spanning tree.

29. Let G be a graph with n vertices, where $n \geq 2$. Then prove that G has at least two vertices whi are not cut vertices.

30. Let G be a graph in which the degree of every vertex is at least two. Then prove that G contains cycle.

31. Prove that an edge e of graph G is a bridge if and only if e is not a part of any cycle in G.

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32. Prove that a simple graph G is Hamiltonian if and only if its closure $c(G)$ is Hamiltonian.

33. Let G be a simple planar graph with less than 30 edges. Prove that G has a vertex v with $d(v) \leq 4$.

(5 × 6 = 30 marks)

Section D

Answer any **one** *question.*
The question carries 14 *marks.*

34. (a) Define bipartite graph.

(b) Let G be a non-empty graph with at least two vertices. Prove that G is bipartite if and only if it has no odd cycles.

35. Let G be a simple graph with n vertices. Then prove that the following statements are equivalent

(i) G is a tree.

(ii) G is an acyclic graph with $n - 1$ edges.

(iii) G is an connected graph with $n - 1$ edges.

36. (a) If G is a simple planar graph then prove that G has a vertex v of degree less than six.

(b) Prove that K_5 is non-planar graph.

(c) Prove that $K_{3,3}$ is non-planar graph.

[1 × 14 = 14 marks]