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C 80711 (Pages : 2) Name.....................................

Reg. No.....................................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCSS)

Mathematics

MT 4E 10—ADVANCED OPERATIONS RESEARCH

Time : Three Hours Maximum : 36 Weightage

Part A

Answer **all** *the questions.*
Each question has weightage 1.

1. Distinguish between linear programming and non-linear programming.
2. Define convex function and give an example for a convex function.
3. Define Lagrange multipliers.
4. Write the general form of a convex programming problem.
5. State the condition under which the function $F(X, Y)$ has a saddle point.
6. Write the Kuhn-Tucker conditions for the saddle points of a function.
7. Write the general form of a qudratic programming problem.
8. When do we say that a programming problem is separable ?
9. Write the general form of geometric programming problem.
10. Write the model of a serial multistage problem.
11. When do we say that an optimization problem is decomposable ?
12. Define decision variables in dynamic programming.
13. What is meant by return function ? Illustrate using an example.
14. Describe the forward recursion procedure.

(14 × 1 = 14 weightage)

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Part B

Answer any seven questions.
Each question has weightage 2.

15. Mark on graph the feasible solutions of $(x_1 - 1)(x_2 - 1) \leq 1, x_1 + x_2 \geq 6, x_1 \geq 0, x_2 \geq 0.$

16. If $F(X, Y)$ has a saddle point (X_0, Y_0) for every $Y \geq 0$, then with usual notations prove th $G(X_0) \leq 0, Y_0' G(X_0) = 0.$

17. Describe how the Kuhn-Tucker theorem is derived from a convex programming problem.

18. Write the Kuhn-Tucker conditions to minimize $f = x_1^2 + x_2^2$ subject to $g = (x_1 - 1)^2 - x_2^2 \geq 0.$

19. Discuss the primal-dual concept in geometric programming.

20. Maximize x^4 subject to $-\frac{1}{2} \leq x \leq 1.$

21. Justify the name geometric programming to problems involving polynomials.

22. Describe a minimum path problem in Dynamic programming.

23. Describe the computational economy in Dynamic programming.

24. What is the serial multistage model in dynamic programming ? Discuss.

(7 × 2 = 14 weight

Part C

Answer any two questions.
Each question has weightage 4.

25. Minimize $f = 2x_1 - 3x_2$ subject to $4x_1^2 + 9x_2^2 \leq 36, x_1 \geq 0, x_2 \geq 0$ using separable programm technique.

26. By the method of quadratic programming, minimize

$$-6x_1 + 2x_1^2 - 2x_1x_2 + 2x_2^2 \text{ subject to } x_1 + x_2 \leq 2, x_1 \geq 0, x_2 \geq 0.$$

27. Use geometric programming to find the dimensions of a rectangle of maximum area inscribe circle of radius r.

28. Maximize $\sum_{n=1}^{4} (4u_n - nu_n)^2$ subject to $\sum_{n=1}^{4} u_n = 10, u_n \geq 0.$

(2 × 4 = 8 weigh

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C 80715

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Name.................................

Reg. No..............................

FOURTH SEMESTER M.Sc. DEGREE EXAMINATION, MARCH 2020

(CUCSS)

Mathematics

MT4 E14—DIFFERENTIAL GEOMETRY

me : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Describe the level set at $c = 2$ for the function $f(x_1, x_2) = x_1 + x_2$.

2. Sketch the vector field on $\mathbb{R}^2$ given by $X(p) = (1, 0)$.

3. Show that the plane curve $x_1 + 2x_2 + 3x_3$ is a level set of some function f.

4. Give an orientation on the 2–sphere.

5. Define Gauss map.

6. Give an example of a vector field along a parametrized curve.

7. Let α be a geodesic on S. Show that $\dot{\alpha}' = 0$ along α.

8. Define parallel vector field along a parametrized curve α.

9. Show that $\nabla_{v+w} f = \nabla_v f + \nabla_w f$ for all smooth functions f.

10. Find $L_p(v)$ for $p = (1, 0, 0)$ and $v = (2, 1, 0)$ on the sphere $x_1^2 + x_2^2 + x_3^2 = 1$.

11. Find the length of the parametrized curve $\alpha : [0, \pi] \to \mathbb{R}^2$ given by $\alpha(t) = (\cos t, \sin t)$.

12. Find the Gaussian curvature of an n–surface S at p where the principal curvatures are $1, 1/2, 1/3$..

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13. Give an example of a parametrized 1–surface.

14. Describe the stereographic projection from a sphere to a plane.

(14 × 1 = 14 weighta

Part B

Answer any **seven** *questions.*
Each question carries weightage 2.

15. Sketch the gradient field of the function $f(x_1, x_2) = x_1^2 + x_2^2$.

16. Describe the tangent space of the unit circle at $p = (1, 0)$.

17. Find the spherical image of the 2–sphere $x_1^2 + x_2^2 + x_3^2 = 2$.

18. With the usual notations prove that $(f\dot{X}) = f'X + f\dot{X}$.

19. Let S be an n–surface. Prove that if S contains a line segment $\alpha(t) = p + tv$ for $t \in [0, 1]$ then a geodesic on S.

20. Let X' denote the covariant derivative of a vector field X along a parametrized Curve α. S that $(X + Y)' = X' + Y'$

21. Find the curvature k of the plane curve $\frac{x_1^2}{3} + \frac{x_2^2}{5} = 1$.

22. Let $k(p)$ be the curvature of a palne curve at p and v be a non-zero vector tangent to the curve at p. Show that $k(p) = L_p(v) \cdot v / \| v^2 \|$.

23. Let N (v) be the normal section of an n—surface determined by a unit vector v. Show that N isomorphic to a copy of $\mathbb{R}^2$.

24. Described the parametrized torus in $\mathbb{R}^4$.

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Part C

Answer any **two** *questions.*
Each question carries weightage 4.

25. (a) Define integral curve of a vector field.

(b) Show that for the vector field $X(x_1, x_2) = (-x_2, x_1)$, the parametrized curve $\alpha(t) = (\cos t - \sin t, \cos t + \sin t)$ in an integral curve on it.

(c) Find an integral curve for the vector field given by $X(x_1, x_2) = (x_2, x_1)$.

26. Let S be an n–surface in $\mathbb{R}^{n+1}$ and let X be a smooth tangent vector field on S and let $p \in S$. Show that there exists an open interval I containing 0 and a parametrized curve $\alpha : I \to S$ such that :

(a) $\alpha(0) = p$; and

(b) $\dot{\alpha}(t) = X(\alpha(t))$ for $t \in I$.

27. Let S be an n–surface in $\mathbb{R}^{n+1}$ and $\alpha : I \to S$ be a prarametrized curve in S. Let X be a vector field along α. Prove that :

(a) X is parallel along α if and only if X satisfies the differential equation

$$\dot{X}(t) + \left(X(t) \cdot \dot{N}(\alpha(t))\right) . N(\alpha(t)) = 0 \text{ for all } t.$$

(b) If X is a solution of the above differential equation then X is tangent to S along α.

28. (a) Define second fundamental form of an oriented n–surface.

(b) Let S be a compact n–surface in $\mathbb{R}^{n+1}$. Snow that there exists $p \in S$ such that the second fundamental form at p is definite.

(2 × 4 = 8 weightage)

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