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Name...

Reg. No......................................

FOURTH SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2024

(CBCSS)

Mathematics

MTH4E11—GRAPH THEORY

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Prove that every connected graph contains a spaning tree.
2. If G is a connected graph then prove that $\epsilon \geq \upsilon - 1$.
3. Draw a simple graph G with $\kappa < \kappa' < \delta$.
4. What is the maximum number of perfect matching in a tree.
5. Define Ramsey numbers.
6. Give an example of a graph with $\chi < \Delta + 1$.
7. Prove that an inner bridge that avoids outer bridge is transferable.
8. If G is non planar, prove that every subdivision of G is non planar.

($8 \times 1 = 8$ weightage)

Part B

Answer any **two** *questions from each unit.*
Each question carries a weightage 2.

Unit I

9. Show that an edge e of a graph G is a cut edge of G if and only if e is not contained in no cycle of G.
10. If G isa block with $\upsilon \geq 3$, then show that any two edges of G lie on a common cycle.
11. If G isa simple graph with $\upsilon \geq 3$ and $\delta \geq \frac{\upsilon}{2}$, then prove that G is hamiltonian.

Turn over

Unit II

12. If G is bipartite, prove that $\chi' = \Delta$.

13. Prove that every 3–regular graph without cut edges has a perfect matching.

14. Prove that, in a bipartite graph G with $\delta > 0$, the number of vertices in a maximum ind set is equal to the number of edges in a minimum edge covering.

Unit III

15. Prove that every planar graph is 5-vertex clourable.

16. If G is simple, prove that $\pi_K(G) = \pi_K(G-e) - \pi_K(G.e)$ for any edge e of G.

17. Prove that a digraph D contains a directed path of length $\chi - 1$.

(6 × 2 = 12 w

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Prove that a graph is hamiltonian if and only if its closure is hamiltonian.

19. Prove that a graph G has a perfect matching if and only if $o(G-S) < |S|$ for all $S \subset V$.

20. If G is 4—chromatic, then prove that G contains a subdivision of K_4.

21. If G is a connected simple graph and is neither an odd cycle nor a complete graph, pr $\chi \le \Delta$.

(2 × 5 = 10 we