87828

D 11675 **(Pages : 3)**

Name.......................................

Reg. No..................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Prove that if $A \in L(\mathbb{R}^n, \mathbb{R}^m)$, then $\| A \| < \infty$ and A is a uniformly continuous mapping of $\mathbb{R}^n$ into $\mathbb{R}^m$.

2. Show that $\det [A] = 0$ if $[A]$ is $n \times n$ matrices having two equal columns.

3. Define a parametrized curve. Find the parametrization for the level curve $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

4. Verify whether $\sigma(u, v) = (u, v^2, v^3)$; $u, v \in \mathbb{R}$ a regular surface patch or not.

Turn over

5. Find the equation of the tangent plane of the surface patch $\sigma(u, v) = (u, v, u^2 - v^2)$ at the poi

$(1, 1, 0)$.

6. Show that $x^2 + y^2 = z^2$ is a smooth surfaces.

7. Calculate the first fundamental forms of the surface $\sigma(u, v) = (\cosh u, \sinh u, v)$.

8. Show that every local isometry is conformal. Give an example of a conformal map that is not a lo isometry.

(8 × 1 = 8 weighta

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

UNIT 1

9. Let Ω be the set of all invertible linear operator on $\mathbb{R}^n$, show that :

   (a) If $A \in \Omega, B \in L(\mathbb{R}^n)$, and $\| B - A \|, \| A^{-1} \| < 1$, then $B \in \Omega$.

   (b) Ω is an open subset of $L(\mathbb{R}^n)$, and the mapping $A \to A^{-1}$ is continuous on Ω.

10. Show that if X is a complete metric space, and if φ is a contraction of X into X, then there exis and only one $x \in X$ such that $\varphi(x) = x$.

11. Show that a linear operator A on $\mathbb{R}^n$ is invertible if and only if $\det[A] \neq 0$.

UNIT 2

12. If $\gamma(t)$ be a regular curve in $\mathbb{R}^3$, then show that its curvature is $\kappa = \dfrac{\| \ddot{\gamma} \times \dot{\gamma} \|}{\| \dot{\gamma} \|^3}$.

13. Let γ be a unit-speed curve in $\mathbb{R}^3$ with constant curvature and zero torsion. Then, show th a parametrization of part of a circle.

14. Suppose that two smooth surfaces S and $\tilde{S}$ are diffeomorphic and that S is orientable. Prove that $\tilde{S}$ is orientable.

UNIT 3

15. Show that any tangent developable is locally isometric to a plane.

16. Calculate the Gaussian curvature of $\sigma(u, v) = (f(u)\cos v, f(u)\sin v, g(u))$ where $f > 0$ and $\dot{f}^2 + \dot{g}^2 = 1$.

17. Calculate the principal curvatures of the catenoid $\sigma(u, v) = (\cosh u \cos v, \cosh u \sin v, u)$.

(6 × 2 = 12 weightage)

Part C

Answer **two** *questions.*
Each question has weightage 5.

18. State and prove the Implicit function theorem.

19. Let $\gamma(s)$ and $\tilde{\gamma}(s)$ be two unit-speed curves in $\mathbb{R}^3$ with the same curvature $\kappa(s) > 0$ and the same torsion $\tau(s)$ for all s. Then, there is a direct isometry M of $\mathbb{R}^3$ such that $\tilde{\gamma}(s) = M(\gamma(s))$ for all s. Further, if k and t are smooth functions with $k > 0$ everywhere, there is a unit-speed curve in $\mathbb{R}^3$ whose curvature is k and whose torsion is t.

20. Let S and $\tilde{S}$ be surfaces and let $f : S \to \tilde{S}$ be a smooth map. Then, prove that f is a local diffeomorphism if and only if, for all $p \in S$, the linear map $D_p f : T_p S \to T_{f(p)} \tilde{S}$ is invertible.

21. A local diffeomorphism $f : S_1 \to S_2$ is conformal if and only if there is a function $\lambda : S_1 \to \mathbb{R}$ such that $f^* \langle v, w \rangle_p = \lambda(p) \langle v, w \rangle_p$ for all $p \in S_1$ and $v, w \in T_p S_1$.

(2 × 5 = 10 weightage)