D 11678 (Pages : 4) Name.......................................

Reg. No.................................

THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

General Instructions (Not applicable to SDE/Private Students)

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Solve $u_x = 1$ subject to the condition $u(0, y) = g(y)$.

2. For the equation $u_{xx} + xu_{yy} = 0, x > 0$ find a canonical transformation $q = q(x, y), r = r(x, y)$ and the corresponding canonical form.

3. Derive the general solution of one dimensional wave equation.

4. Show that the only possible value for the eigen value problem $\frac{d^2X}{dx^2} + \lambda X = 0, 0 < x < L, X(0) = X(L) = 0$ are positive real numbers.

5. State mean value principle property of harmonic functions.

Turn over

D 1

6. Show that the Dirichlet problem has atmost one solution in a smooth domain.

7. Explain the volterra equation of third kind ? Can the same be transformed to that of kind ? Justify.

8. If $y'' = F(x)$ and y satisfies the initial condition $y(0) = y_0, y'(0) = y'_0$. Show

$$y(x) = \int_0^x (x-\xi) F(\xi)\, d\xi + y'_0 x + y_0.$$

(8 × 1 = 8 w

Part B

Answer **six** *questions choosing* **two** *from each unit.*
Each question has weightage 2.

Unit 1

9. Solve the equation $-yu_x + xu_y = u$ subject to the initial condition $u(x, 0) = \psi(x)$.

10. Show that the Cauchy problem $u_x + u_y = 1, u(x. x) = x$ has uniquely many solutions

11. Explain and justify the well posedness of Cauchy problem.

Unit 2

12. Solve $u_t - 17\, u_{xx} = 0, 0 < x < \pi, t > 0$

$$u(0, t) = u(\pi, t) = 0, t \geq 0$$

$$u(x, 0) = f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{\pi}{2} \\ 2, & \frac{\pi}{2}, \leq x \leq \pi \end{cases}.$$

13. Show that the Neumann problem for the vibrating string $u_{tt} - c^2\, u_{xx} = F(x, t)$ subject to the conditions

$$u_x(0, t) = a(t), u_x(L, t) = b(t), t \geq 0$$

$$u(x, 0) = f(x), 0 \leq x \leq L$$

$u_t(x, 0) = g(x), 0 \leq x \leq L$, has unique solution.

14. State and prove the weak maximum principle.

UNIT 3

15. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real.

16. Show that $y'' + Ay' + By = 0$, $y(0) = y(1) = 0$, where A and B are constants, leads to the equation

$$y(x) = \int_0^1 k(x, \xi)\, y(\xi)\, d\xi, \text{ where } k(x, \xi) = \begin{cases} B\xi(1-x) + Ax - A, & \xi < x \\ Bx(1-\xi) + Ax, & \xi > x \end{cases}.$$

17. Formulate the integral equation corresponding to the differential equation $x^2y'' + xy' + (\lambda x^2 - 1)y = 0$.

(6 × 2 = 12 weightage)

Part C

Answer **two** *questions.*
Each question has weightage 5.

18. Use the method of characteristic strips to solve the non-linear eikonal equation $p^2 + q^2 = n_0^2$ subject to the condition $u(x, 2x) = 1$.

19. For the problem $u_{tt} - 4u_{xx} = 0, -\infty < x < \infty, t > 0$ with initial conditions

$$u(x, 0) = f(x) = \begin{cases} 1 - x^2, & |x| \le 1 \\ 0, & \text{otherwise} \end{cases}$$

$$u_t(x, 0) = \begin{cases} 4, & 1 \le x \le 2 \\ 0, & \text{otherwise} \end{cases}.$$

a) Find $u(x, 1)$.

b) Find $\lim_{t \to \infty} u(5, t)$.

Turn over

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c) Find the set of all points where the solution is singular.

d) Find the set of all points where the solution is continuous.

20. a) Using the separation of variables method find a (formal) solution of a vibrating str
fixed ends :

$$u_{tt} - c^2 u_{xx} = 0, \; 0 < x < L, \; 0 < t$$
$$u(0, t) = u(L, t) = 0, \; t \geq 0$$
$$u(x, 0) = f(x) \; 0 \leq x \leq L,$$
$$u_t(x, 0) = g(x) \; 0 \leq x \leq L.$$

(b) Prove that the above solution can be represented as a superposition of a forward and
wave.

21. Determine the resolvent kernel of $y(x) = 1 + \lambda \int_0^1 (1 - 3x\xi)\, y(\xi)\, d\xi$ where $k(x, \xi) = 1 -$

(2 × 5 = 10