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**THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021**

[November 2020 for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

**General Instructions (Not applicable to SDE/Private Students)**

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

**Part A**

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n, a \in \mathbb{C}$.
2. What is Mobius transformation ? Is the mapping $T(z) = \bar{z}$ a Mobius transformation. Justify your claim.
3. Find the image of the following points in the complex plane 0, $1 + i$, $3 + 2i$ on the unit sphere.
4. Show that $\lim n^{\frac{1}{n}} = 1$.
5. If $\sum a_n$ converges absolutely then prove that $\sum a_n$.
6. Find the image of the lines $x = \alpha$ under the mapping $\cos z$.

**Turn over**

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7. Determine the type of singularity of $f(z) = \frac{\cos z}{z}$ at $z = 0$.

8. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

(8 × 1 = 8 weig

## Part B

*Answer* **six** *questions choosing two from each unit.*
*Each question has weightage 2.*

### Unit I

9. Define $\gamma : [0, 2\pi] \to C$ by $\gamma(t) = \exp(int)$ where $n$ is some integer (positive, negative or zero

that $\int_{\gamma} \frac{1}{z} dz = 2\pi in$.

10. Prove that there is no branch of the logarithm defined on G = C – {0}.

11. Prove that a Mobius transformation carries circles into circles.

### Unit II

12. Show that the Integral Formula follows from Cauchy's Theorem.

13. If $\gamma : [0,1] \to C$ is a closed rectifiable curve in G such that $\gamma \sim 0$ then prove that $n(\gamma ; w$
in C – G.

14. Find $\int_{\gamma} z^{\frac{-1}{2}} dz$ where $\gamma$ is the upper half of the unit circle from +1 to – 1.

### Unit III

15. If $\gamma$ is piecewise differentiable and $f : [a,b] \to C$ is continuous then prove that $\int_a^b f d r =$

16. If G is a region and suppose that $f : G \to C$ is analytic and $a \in G$ such that $|f(a)| \leq |f$
$z$ in G. Show that either $f(a) = 0$ or $f$ is constant.

17. If $\gamma$ is a closed rectifiable curve in G such that $\gamma \sim 0$ then $n(\gamma ; w) = 0$ for all $w$ in C –

(6 × 2 = 12

## Part C

*Answer* **two** *questions.*
*Each question has weightage 5.*

18. Let $f(z) = \sum_{n=0}^{\infty} a_n (z = a)^n$ have radius of convergence R > 0. Then :

(a) For each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)a_n(z-a)^{n-k}$ has radius of convergence R.

(b) The function $f$ is infinitely differentiable on B $(a$,R) and, furthermore, $f^k(z)$ is given by $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1)a_n(z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| < \text{R}$.

(c) For $k \geq 1, a_n = \frac{1}{n!} f^n(a)$.

19. State and prove open mapping theorem.

20. State and prove Goursat's Theorem.

21. (a) Evaluate the integral $\int_{\gamma} \frac{e^z - e^{-z}}{z^n} dz$ where $n$ is a positive integer and $\gamma(t) = e^{it}, 0 \leq t \leq 2\pi$.

(b) Prove the following Minimum Principle. If $f$ is a non-constant analytic function on a bounded open set G and is continuous on $\text{G}^-$, then either $f$ has a zero in G or $|f|$ assumes its minimum value on $\partial\text{G}$.

(2 × 5 = 10 weightage)