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Name.....................................

Reg. No.....................................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CUCSS)

Mathematics

MT 3C 15—PDE AND INTEGRAL EQUATIONS

(2016 Admissions)

me : Three Hours

Maximum : 36 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Find the partial differential equation of all spheres of radius r, having center in the xy plane.

2. Show that $(x-z)(y-z)=1$ is a singular integral of $z = px + qy - 2\sqrt{pq}$.

3. Verify that the Paffian differential equation $yzdx + 2xzdy - 3xydz = 0$ is integrable and find the corresponding integrals.

4. Find the complete integral of $4(1+z^3) = 9z^4pq$.

5. Determine the characteristic curve for solving the equation $z_x - zz_y + z = 0$ for every y and $x > 0$ with the initial conditions $x_0 = 0, y_0 = s, z_0 = -2s, -\infty < s < \infty$.

6. What is the range of influence of the point ?

7. Describe the second and third boundary value problem of the Laplace equation.

8. Show that the solution of the Neumann problem is unique up to the addition of constant.

9. Classify the partial differential equation $xy\frac{\partial^2 z}{\partial x^2} - \left(x^2 - y^2\right)\frac{\partial^2 z}{\partial x \partial y} - xy\frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x}y - \frac{\partial z}{\partial y}x = x^2 - y^2$.

Turn over

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10. Show that $y(x) = \cos(2x)$ is the solution of the integral equation $y(x) = \cos(x) + 3\int_0^\pi K(x, \xi)\, y(\xi)$

where $K(x, \xi) = \begin{cases} \sin(x)\cos(\xi), & 0 \le x \le \xi \\ \cos(x)\sin(\xi), & \xi \le x \le \pi \end{cases}$.

11. Define different types of kernel of an integral equation with an example.

12. Find the eigenvalue and the eigen functions of the homogeneous integral equa

$y(x) = \lambda \int_0^1 \sin(\pi x)\cos(\pi\xi)\, d\xi$.

13. Find the solution of the integral equation $g(x) = x + \int_0^1 x\xi^2 g(\xi)\, d\xi$.

14. Determine the resolvent kernel associated with $k(x, \xi) = e^{x+\xi}$ in (0, 1), in the form of a po series in λ.

(14 × 1 = 14 weighta

Part B

Answer any **seven** *questions.*
Each question carries 2 *weightage.*

15. Find the general integral of the equation $(y + z)p + (z + x)q = x + y$.

16. Using Charpit's method find the complete integral of $(p^2 + q^2) = z^2(x + y)$.

17. Solve the equation $p^2x + q^2y = z$ by Jacobi's method.

18. Reduce the equation $y^2 \dfrac{\partial^2 z}{\partial x^2} + x^2 \dfrac{\partial^2 z}{\partial y^2} = 0$ into canonical form.

19. Show that the surfaces $x^2 + y^2 + z^2 = c^{\frac{2}{3}}$ can form a family of equipotential surfaces, and find t general form of the corresponding potential function.

20. State the one dimensional vibration of an infinite string and derive D'Alambert's solution.

21. State the Heat conduction problem for finite rod of length l with the initial temperature $f(x)$ with heat source $F(x, t)$. Show that if the solution exists then it is unique.

22. Transform the boundary value problem $\frac{d^2y}{dx^2} + xy = 1, y(0) = y(1) = 0$ into an integral equation.

23. Show that the characteristic functions of the symmetric kernel corresponding to distinct characteristic numbers are orthogonal.

24. Solve $y(x) = \sin(x) + 2\int_0^x e^{x-\xi}\, y(\xi)\, d\xi$ by iterative method.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*

Each question carries 4 weightage.

25. Define compatibility of system of first order PDE and establish the necessary and sufficient condition to obtain the one parameter family of common solutions.

26. Determine the characteristic of the equation $pq = z$ which passes through the parabola $x = 0$, $y^2 = z$.

27. Using Riemann method find the solution of the non-homogeneous wave equation

$$\frac{\partial^2 z}{\partial x^2} - \frac{1}{c^2}\frac{\partial^2 z}{\partial t^2} + f(x, t) = 0$$

with the initial condition $z(x, 0) = f(x)$, $z_t(x, 0) = g(x)$.

28. Show that the integral equation $y(x) = f(x) + \frac{1}{\pi}\int_0^{2\pi} \sin(x+\xi)\, y(\xi)\, d\xi$ possesses no solution for $f(x) = x$, but that it possesses infinitely many solutions when $f(x) = 1$.

(2 × 4 = 8 weightage)