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91039 (Pages : 3) Name..............................

Reg. No..............................

THIRD SEMESTER M.A./M.Sc./M.Com. DEGREE (REGULAR) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admissions)

me : Three Hours Maximum : 30 Weightage

General Instructions

. *In cases where choices are provided, students can attend* **all** *questions in each Section/Part.*

. *The minimum number of questions to be attended from the Section/Part shall remain same.*

. *There will be an overall ceiling for each Section/Part that is equivalent to maximum weightage of the Section/Part.*

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Explain the concept of radius of convergence of a power series ? Find the radius of convergence of the power series $\sum_{n=0}^{\infty} a^n z^n$.

2. Prove that $\sum a_n$ an converges if $\sum a_n$ converges absolutely.

3. Let $\gamma:[0,2\pi]\to C$ and define $f(z)=\frac{1}{z}$; for $z\neq 0$ find $\int_{\gamma} f(z)\,dz$.

4. Prove that a Mobius transformation is the composition of translation, dilation and the inversion.

5. Identify the type of singularity of the function $\frac{\sin z}{z}$ at $z=0$.

Turn over

6. If $\sum a_n$ and $\sum b_n$ converges absolutely then prove that $\sum c_n$, where $c_n = \sum_{n=0}^{n} a_k b_{n-k}$ conver

absolutely.

7. Show that $f(z) = \tan z$ is analytic in C except for simple poles at $z = \pi 2 + n\pi$, for each integer. Determine the singular part of f at each of these poles.

8. Define the residue of the function $f(z)$ at the singularity a.

(8 × 1 = 8 weighta

Part B

Answer any **six** *questions.*
Each question has weightage 2.

9. If G is open and connected and $f: G \to C$ is differentiable with $f'(z) = 0$ for all $z \in G$, then pro that f is constant.

10. Show that the function $f(z) = \sqrt{|xy|}$ is not analytic at the origin even though Cauchy-Riema equations are satisfied.

11. Find the image of $\{z : \text{Re}(z) = 0\}$ and $\{z : \text{Im}\, z = \pi/2\}$ under the exponential function.

12. State and prove Liouville's theorem.

13. Give the power series expansion of $\log z$ about $z = i$ and find its radius of convergence.

14. Let γ be the closed polygon $[1 - i, 1 + i, -1 + i, -1 - i, 1 - i]$. Find $\int_\gamma \frac{1}{z}\, dz$.

15. If $z = a$ is an isolated singularity of f and $f(z) = \sum_{-\infty}^{\infty} a_n (z - a)^n$ be the Laurent Expansion i

$ann\,(a\,; 0, R)$ then prove that $z = a$ is a removable singularity if and only if $a_n = 0$ for $n \leq -1$.

6. Evaluate $\int_0^{\infty} \frac{x^2 dx}{x^4 + x^2 + 1}$.

7. Find the Laurent series expansion of $e^{\frac{1}{z}}$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

8. (a) Let u and v be real-valued functions defined on a region G and suppose that u and v have continuous partial then prove that $f : G \to C$ defined by $f(z) = u(z) + iv(z)$ is analytic if and only if it satisfies the C-R equations.

(b) Show that the real part of the function $\frac{1}{z^2}$ is always positive.

19. (i) Prove that a Mobius transformation carries circles into circles.

(ii) If z_1, z_2, z_3, z_4 are four distinct points in C_{∞}, then prove that their cross ratio is real if and only if all four points lie on a circle.

20. (a) State and prove general form of Cauchy's theorem.

(b) Let γ and σ be the two polygons $[1, i]$ and $[1, 1+i, i]$. Express γ and σ as paths and calculate

$$\int_{\gamma} f \text{ and } \int_{\sigma} f \text{ where } f(z) = |z|^2.$$

21. State and prove the argument principle.

(2 × 5 = 10 weightage)