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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2022

[November 2021 session for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 14—PDE AND INTEGRAL EQUATIONS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Solve $u_x + u_y = 2$ subject to the condition $u(x, 0) = x^2$.

2. Describe the condition for a first order quasilinear partial differential equation to have a unique solution.

3. Let $u(x, t)$ be a solution for wave equation $u_{tt} - c^2 u_{xx} = 0$ which is defined in the whole plane. Assume that u is constant on the line $x = 2 + ct$. Prove that $u_t + cu_x = 0$.

4. Show that $\lambda = \left(\frac{n\pi}{L}\right)^2$, $n = 1, 2, 3....$ are the only possible values for the eigen value problem

$$\frac{d^2X}{dx^2} + \lambda X = 0, 0 < x < L, X(0) = X(L) = 0.$$

5. State the strong maximum principle of harmonic functions.

6. Consider the Dirichlet problem in a bounded domain $\Delta u = f(x, y), (x, y) \in D, u(x, y) = g(x, y), (x, y) \in \delta D$. Then proc=ve that the problem has atmost one solution in $C^2 D \cap C(\bar{D})$.

Turn over

7. Discuss the Green's function for the differential equation $\left(\frac{d}{dx}\left(p\frac{d}{dx}\right)q\right)y+\varphi(x)=0.$

8. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel a real.

(8 × 1 = 8 weigh

Part B

Answer any **six** *questions choosing two from each unit.*
Each question has weightage 2.

UNIT 1

9. Solve the pde $u_x + 3y^{2\,3}u_y = 2$ subject to the initial condition $u(x,1) = 1 + x$.

10. Find the canonical form of $x^2u_{xx} - 2xyu_{xy} + y^2u_{yy} + xu_x + yu_y = 0$ and find the general sol on the half plane $x > 0$.

11. Describe the Cauchy problem for non homogeneous wave equation. Also show that it has a one solution.

UNIT 2

12. Solve $u_t - 17\,u_{xx} = 0, 0 < x < \pi, t > 0$

$u(0,t) = u(\pi,t) = 0, t \geq 0$

$$u(x,0) = f(x) = \begin{cases} 0, 0 \leq x \leq \frac{\pi}{2} \\ 2, \frac{\pi}{2}, \leq x \leq \pi \end{cases}.$$

13. Show that the Neumann problem for the vibrating string $u_{tt} - c^2u_{xx} = F(x,t), 0 < x < L$ subject to the conditions

$u(0,t) = a(t), u(L,t) = b(t), t \geq 0$

$u(x,0) = f(x), 0 \leq x \leq L$

$u_t(x,0) = g(x), 0 \leq x \leq L$, has unique solution.

14. Prove that the function u in C^2 (D) satisfying the mean value property at every point D will be harmonic in D.

UNIT 3

15. Transform the integral equation $y'' + \lambda y = 0$ with $y(0) = 0, y(l) = 0$ to an integral equation.

16. Let $y_m(x), y_n(x)$ are characteristic functions corresponding to distinct characteristic values λ_m, λ_n respectively of a homogeneous Fredholm equation $y(x) = \lambda \int_a^b k(x, \xi) y(\xi) d\xi$ with symmetric kernel. Show that $y_m(x)$ and $y_n(x)$ are orthogonal.

17. Show that the differential equation $y'' + xy = 1, y(0) = 0 = y(1)$ can be written as the integral equation $y = \lambda \int_0^1 G(x, \xi) y(\xi) d\xi + \frac{x(x-1)}{2}$, where $G(x, \xi) = \begin{cases} \xi(x-1), \xi < x \\ x(\xi - 1), \xi > x \end{cases}$.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. Let $u(x, t)$ be the solution of the Cauchy problem $u_{tt} - 9u_{xx} = 0, -\infty < x < \infty, t > 0$.

$$u(x, 0) = f(x) = \begin{cases} 1, |x| \le 2 \\ 0, |x| > 2 \end{cases}$$

$$u_t(x, 0) = g(x) = \begin{cases} 1, |x| \le 2 \\ 0, |x| > 2 \end{cases}$$

a) Find $u(0, 1/6)$;

b) Discuss the large time behavior of the solution;

c) Find the maximal value of $u(x, t)$ and the points where this maximum is achieved ; and

d) Find all points where $c \in C^2$.

Turn over

19. Find the formal solution of the problem.

$u_{tt} - u_{xx} = 0, 0 < x < \pi, t > 0$

$u(0, t) = u(\pi, t) = 0, t \geq 0$

$u(x, 0) = f(x) = \sin^3 x, 0 \leq x \leq \pi$

$u_t(x, 0) = g(x) = \sin 2x, 0 \leq x \leq \pi.$

20. Let u be the harmonic function in the unit square satisfying the Dirichlet cond $u(x, 0) = 1 + \sin \pi x, u(x, 1) = 2, u(0, y) = u(1, y) = 1 + y$. Represent u as a sum of har polynomial and a harmonic function $v(x, y)$ that satisfies the compatibility condition.

21. For the integral equation $y = \lambda \int_0^1 (1 - 3x\xi) y(\xi) d\xi + F(x)$, find the characteristic numbe characteristic functions. Also show that $y(x) = F(x) + a_1(1 - x) + a_2(1 - 3x)$.

(2 × 5 = 10 wei