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# THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2022

[November 2021 session for SDE/Private Students]

(CBCSS)

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

## Part A

*Answer* **all** *questions.*
*Each question has weightage* 1.

1. If the function $f$ defined on a domain G is differentiable at a point $a$ in G then prove that $f$ is continuous at $a$.

2. Prove that $|\exp z| = \exp(\operatorname{Re} z)$.

3. Prove that $u(x, y) = \log\left(x^2 + y^2\right)$ is harmonic on $G = C - \{0\}$.

4. Let $\gamma$ be the closed polygon $[1 - i, 1 + i, -1 + i, -1 - i, 1 - i]$. Find $\int_\gamma \frac{1}{z+2}\, dz$.

5. Establish Cauchy's Estimate.

6. Evaluate $\int_C \frac{2z^2 + z}{z^2 - 1}\, dz$ where C is $|z| = 1$.

7. If G is an open set which is $a$-star shaped and if $\gamma_0$ is a curve which is constantly equal to $a$ then prove that every closed rectifiable curve in G is homo topic to $\gamma_0$.

8. Find the singularities of $f(z) = \frac{\sin z}{z}$ and identify the type of singularities.

(8 × 1 = 8 weightage)

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## Part B

*Answer any* **six** *questions choosing* **two** *from each unit.*
*Each question has weightage* 2.

### UNIT I

9. Prove that $f(z) = |z|^2 = x^2 + y^2$ has a derivative only at the origin.

10. If $u$ is a real-valued function defined on a region then prove that $u$ has a harmonic conjugate i is harmonic.

11. Define Cross ratio. Prove that cross ratio remains invarient under Mobius transformation.

### UNIT II

12. Let $\gamma : [a, b] \to \mathrm{R}$ be non-decreasing. Show that $\gamma$ is of bounded variation and $\mathrm{V}(\gamma) = \gamma(b) - \gamma($

13. If $\gamma : [0, 1] \to \mathrm{C}$ is a closed rectifiable curve and $a \notin \{\gamma\}$ then prove that $\frac{1}{2\pi i} \int_\gamma \frac{dz}{z - a}$ is an integ

14. State and prove fundamental theorem of algebra.

### UNIT III

15. Find the image of $\{z : \mathrm{Re}\, z < 0, |\mathrm{Im}\, z| < \pi\}$ under the exponential function.

16. If $f$ has an essential singularity at $z = a$ then prove that for every $\delta > 0$; $f[ann(a; 0; \delta)] = \mathrm{C}$

17. Prove that the function $f : [a, b] \to \mathrm{R}$ is convex iff the set $\mathrm{A} = \{(x, y) : a \le x \le b \text{ and } f(x) \le y$ convex.

(6 × 2 = 12 weight

## Part C

*Answer any* **two** *questions.*
*Each question has weightage* 5.

18. If for a given power series $\sum_{n=0}^{\infty} a_n (z-a)^n$ the number $R, 0 \leq R \leq \infty$ is defined by $\frac{1}{R} = \limsup |a_n|^{\frac{1}{n}}$ then prove the following :

(a) If $|z-a| < R$, the series converges absolutely.

(b) If $|z-a| > R$, , the terms of the series become unbounded and so the series diverges.

(c) If $0 < r < R$ then the series converges uniformly on $\{z : |z| \leq r\}$.

19. State and prove open mapping theorem.

20. Show that if $f : C \to C$ is a continuous function such that $f$ is analytic off $[-1, 1]$ then $f$ is an entire function.

21. (i) If $f$ is analytic in a region G and $a$ is a point in G with $|f(a)| \geq |f(z)|$ for all $z$ in G then prove that $f$ must be a constant function.

(ii) If G is a bounded open set in C and suppose $f$ is a continuous function on $G^{-}$ which is analytic in G then prove that $\max\{|f(z)| : z \in G^{-}\} = \max\{|f(z)| : z \in \partial G\}$.

(2 × 5 = 10 weightage)