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(Pages : 4) Name.................................

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THIRD SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Mathematics

MTH3C11—MULTIVARIABLE CALCULUS AND GEOMETRY

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Define linearly independent set of vectors. Illustrate with an example.
2. Give an example of a linear transformation between two vector spaces and find its derivative.
3. Let E be an open set in $\mathbb{R}^n$, and $f : E \to \mathbb{R}^m$. When we say that f is continuously differentiable in E.
4. Define parametrized curve in $\mathbb{R}^n$. Give a parametrization of the astroid $x^{2/3} + y^{2/3} = 1$.
5. Compute the curvature of the curve :

$$\gamma(t) = \left(\frac{4}{5}\cos t, 1 - \sin t, -\frac{3}{5}\cos t\right).$$

6. Describe briefly : Stereographic projection from S^2 to the plane.
7. Let $\sigma(u,v)$ be a surface patch with standard unit normal $N(u, v)$. Then prove that $N_u \cdot \sigma_u = -L$.
8. Prove that the second fundamental form of a plane is zero.

(8 × 1 = 8 weightage)

Turn over

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Part B (Paragraph Type Questions)

Answer any two questions from each module.
Each question carries a weightage 2.

Module I

9. Let r be a positive integer. If a vector space X is spanned by a set of r vectors, then [illegible] $X \leq r$.

10. If X is a complete metric space, and if φ is a contraction of X into X, then prove that there [illegible] one and only one $x \in X$ such that $\varphi(x) = x$.

11. If $A \in L(\mathbb{R}^{n+m}, \mathbb{R}^n)$ and if A_x is invertible, then prove that there corresponds to every $k \in$ [illegible] unique $h \in \mathbb{R}^n$ such that $A(h, k) = 0$.

Module II

12. If the tangent vector of a parametrized curve is constant, then prove that the image of the [illegible] part of a straight line.

13. Let $\gamma : (\alpha, \beta) \to \mathbb{R}^2$ be a unit-speed curve, let $s_0 \in (\alpha, \beta)$ and let φ_0 be such that

$$\dot{\gamma}(s_0) = (\cos \varphi_0, \sin \varphi_0).$$

Then prove that there is a unique smooth function $\varphi : (\alpha, \beta) \to \mathbb{R}$ such that $\varphi(s_0) = \varphi_0$ and [illegible] the equation

$$\dot{\gamma}(s) = (\cos \varphi(s), \sin \varphi(s)).$$

holds for all $s \in (\alpha, \beta)$.

14. Let $f : S_1 \to S_2$ be a diffeomorphism. If σ_1 is an allowable surface patch on S_1, then prove [illegible] $f \circ \sigma_1$ is an allowable surface patch on S_2.

Module III

15. Prove that

$$\| \sigma_u \times \sigma_v \| = (EG - F^2)^{1/2}.$$