3

428480

D 51309

(Pages : 2)

Name.....................................

Reg. No.................................

THIRD SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2023

Mathematics

MTH 3C 12—COMPLEX ANALYSIS

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Describe stereographic projection.
2. Find the radius of convergence of the power series $\sum_{n=0}^{\infty} k^n z^{n^2}, k$ an non-zero integer.
3. Show that the real part of the function $\sqrt{z}$ is always positive.
4. Give the power series expansion of log (z) about $z = i$.
5. Evaluate $\int_{\gamma} \frac{\sin z}{z^3} dz$, where $\gamma(t) = e^{it}, 0 \le t \le 2\pi$.
6. Determine the type of singularity of $f(z) = z \sin\left(\frac{1}{z}\right)$ at $z = 0$.
7. Find the number of zeroes of $z^7 - 4z^3 + z - 1$ enclosed by $|z| = 1$.
8. Show that a function $f : [a,b] \to \mathbb{R}$ is convex iff the set $A = \{(x, y) : a \le x \le b \text{ and } f(x) \le y\}$ is convex.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit.*
Each question carries a weightage of 2.

Unit 1

9. Show that if G is open and connected and $f : G \to \mathbb{C}$ is differentiable with $f'(z) = 0$ for all z in G, then f is constant.
10. Prove that there is no branch of the logarithm defined on $G = \mathbb{C} - \{O\}$.
11. State and prove symmetry principle.

428480

Unit 2

12. Show that if f be analytic in $B(a; R)$ then $f(x) = \sum_{n=0}^{\infty} a_n (z-a)^n$ for $|z-a| < R$, where $a_m = \frac{1}{n!} f^{(n)}(a)$ and this series has radius of convergence $\geq R$.

13. Let $U : \mathbb{C} \to \mathbb{R}$ be a harmonic function such that $U(z) \geq 0$ for all z in C ; prove that U is constant.

14. Suppose that $f : G \to \mathbb{C}$ is analytic and one-one ; show that $f'(z) \neq 0$ for any z in G.

Unit 3

15. State and prove residue theorem.

16. State and prove Rouche's Theorem.

17. State and prove Schwarz's lemma.

(6 × 2 = 12 weightage

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Let $f(z) = \sum_{n=0}^{\infty} a_n (z-a)^n$ have radius of convergence $R > 0$. Then :

(a) For each $k \geq 1$ the series $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1) a_n (z-a)^{n-k}$ has radius of convergence R.

(b) The function f is infinitely differentiable on $B(a, R)$ and, furthermore, $f^k(z)$ is given by $\sum_{n=k}^{\infty} n(n-1)\ldots(n-k+1) a_n (z-a)^{n-k}$ for all $k \geq 1$ and $|z-a| < R$.

(c) For $k \geq 1$, $a_n = \frac{1}{n!} f^n(a)$.

19. State and prove analogue of the Fundamental Theorem of Calculus for line integrals.

20. Show that if γ_0 and γ_1 are two closed rectifiable curves in G and $\gamma_0 \; \gamma_1$ then $\int_{\gamma_0} f = \int_{\gamma_1} f$ every analytic function f on G.

21. Evaluate $\int_0^{\infty} \frac{\log x}{1+x^2} dx$.

(2 × 5 = 10 weightage