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Name.....

Reg. No.....

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION
JUNE 2020

(CUCSS)

Mathematics

MT 2C 09—ODE AND CALCULUS OF VARIATIONS

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A

Answer all *questions.*
Each question carries 1 *weightage.*

1. Write $\sin x$ in power series form.

2. Determine the nature of the point $x = 0$ for the equation $y'' + (\sin x)\, y = 0$.

3. Find the Legendre polynomial $P_2(x)$ using Rodrigue's formula.

4. Write geometric series in the hypergeometric series form.

5. If $J_{\frac{1}{2}}(x)$ be a non-trivial solution of Bessel's equation on the positive x axis, then find the distance between successive zeros of $J_{\frac{1}{2}}(x)$.

6. True *or* False : $P_n(x) \leq P_{n+1}(x)$, $x > 1$, where $P_n(x)$ is the Legendre polynomial of degree n.

7. If $1 + x^{10} = \sum_{n=0}^{10} C_n P_n(x)$, then find C_5, where $P_n(x)$ be the Legendre polynomial of degree $n \geq 0$.

Turn over

8. Find the critical point of the system $\begin{cases} \frac{dx}{dt} = -4x - y \\ \frac{dy}{dt} = x - 2y \end{cases}$.

9. What are the eigenvalues of the problem $y'' + \lambda y = 0, y(0) = y(1) = 0$.

10. What will be nature of critical point, if the roots m_1 and m_2 of the auxiliary equation are conjugat complex but not purely imaginary ?

11. Sketch the phase potrait of the system $\begin{cases} \frac{dx}{dt} = -y \\ \frac{dy}{dt} = x \end{cases}$.

12. For what conditions on a, b, c, the Liapunov function $E(x, y) = ax^2 + bxy + cy^2$ is positi definite ?

13. Write Euler-Lagrange equation.

14. Write the normal form of Bessel's equation.

(14 × 1 = 14 weightag

Part B

Answer any **seven** *questions.*

Each question carries 2 weightage.

15. Find the radius of convergence of the series $\sum_{n=0}^{\infty} \frac{x^n}{n^n}$ using ratio test.

16. Determine the nature of points $x = 0, 1$ for the differential equation $x^2(1-x)y'' + xy' + y = 0$.

17. Verify $\frac{d}{dx}\left[x^p J_p(x)\right] = x^p J_{p-1}(x)$.

18. Find the first approximation of the initial value problem $y' = y, y(0) = 1$.

19. Write the indicial equation for $x\left(1+x^2\right)y''+(\cos x)y'+\left(1-3x+x^2\right)y=0$.

20. Show that $\Gamma(p+1)=p\Gamma(p)$.

21. State Sturm separation theorem.

22. Show that the solution $u(x)$ of the equation $u''(x)+xu(x)=0,\ x>0$ has infinitely many zeros on the positive x-axis.

23. Define geodesic problem.

24. Find the extremal for the functional $\int_{x_1}^{x_2}\left(y^2-y'^2\right)dx$.

(7 × 2 = 14 weightage)

Part C

Answer any two *questions.*

Each question carries 4 weightage.

25. Express $f(x)=x^3+2x^2-x-3$ in terms of Legendre polynomials.

26. If $q(x)<0$ and $u(x)$ is a non-trivial solution of $u''+q(x)u=0$ then prove that $u(x)$ has atmost one zero.

27. Find the general solution of $\begin{cases}\dfrac{dx}{dt}=4x-y\\ \dfrac{dy}{dt}=2x+y\end{cases}$.

28. Find the curve joining the points (x_1, y_1) and (x_2, y_2) that yields a surface of revolution of minimum area when revolved about the x-axis.

(2 × 4 = 8 weightage)