6

C 83064

(Pages : 2)

Name.......................................

Reg. No....................................

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION JUNE 2020

(CBCSS)

Mathematics

MT2C08—TOPOLOGY

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *the questions.*
Each question carries weightage 1.

1. Write the discrete topology on the set S = {1, 2, 3}.
2. Define co-countable topology. Is it a Hausdorff topology ?
3. Give an example of an open set, which is not an open interval in the set of real numbers with usual topology.
4. Prove that every quotient space of a discrete space is discrete.
5. Define weakly hereditary property. Prove that compactness is a weakly hereditary property.
6. If a topological space X is connected, then prove that X cannot be written as the disjoint union of two non-empty closed subsets.
7. Prove that regularity is a hereditary property.
8. Prove that a compact subset in a Hausdorff space is closed.

($8 \times 1 = 8$ weightage)

Part B

Answer any **two** *questions from each unit.*
Each question carries weightage 2.

Unit I

9. Prove that union of two closed sets in a metric space is closed.
10. Prove that second countability is a hereditary property.
11. Prove that composition of two continuous functions is continuous.

Turn over

Unit II

12. Prove that every second countable space is first countable. Is the converse true? Establish.

13. Let X_1, X_2 be connected topological spaces and $X = X_1 \times X_2$ with product topology. Then prove that X is connected.

14. Prove that a space X is locally connected at a point $x \in X$ if and only if for every neighbourhood N of x, the component of N containing x is a neighbourhood of x.

Unit III

15. Prove that all metric spaces are T_0 spaces.

16. Prove that every map from a compact space into a T_2 space is closed.

17. Let A, B be subsets of a space X and suppose there exists a continuous function $f : X \to [0, 1]$ such that $f(x) = 0$ for all $x \in A$ and $f(x) = 1$ for all $x \in B$. Then prove that there exist disjoint open sets U, V such that $A \subset U$ and $B \subset V$.

($6 \times 2 = 12$ weightage)

Part C

Answer any **two** *questions.*

Each question carries weightage 5.

18. (a) Prove that open balls in a metric space are open sets.

    (b) Define scattering topology in the set of real numbers. Prove that in this topology no sequence can converge to an irrational number except an eventually constant sequence.

19. (a) Prove that a second countable space always contains a countable dense subset.

    (b) Define nearness relation on a set X. Prove that there is a one-to-one correspondence between the set of topologies on a set and the set of all nearness relations on that set.

20. (a) Define locally connected space. Write an example for a space which is connected but not locally connected.

    (b) Define path-components of a space X. Prove that a subset C is a path-component of a space X if and only if C is a maximal subset of X with respect to the property of being path-connected.

21. (a) Prove that every regular, Lindeloff space is normal.

    (b) Prove that all T_4 spaces are completely regular.

($2 \times 5 = 10$ weightage)