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Name.......................................

Reg. No...................................

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION, JUNE 2020

(CUCSS)

Mathematics

MT 2C 07—REAL ANALYSIS–II

(2016 Admissions)

Time : Three Hours

Maximum : 36 Weightage

Part A (Short Answer Questions)

Answer **all** *the questions.*
Each question carries a weightage of 1.

1. Show that if $F \in M \wedge m^*(F \Delta G) = 0$, then G is measurable.
2. Show that there exist uncountable sets of zero measure.
3. Show that $f \leq ess^{f,a.e}$.
4. Show that $\int_1^{\infty} \frac{dx}{x} = \infty$.
5. Show that if f and g are measurable, $|f| \leq |g| a.e.$, and g is integrable, then f is integrable.
6. Show that if f is integrable then f is finite-valued a.e.
7. Show that $BV[a,b]$ is a vector space over the real numbers.
8. Show that the Lebesgue set of a function $f \in L(a,b)$ contains any point at which f is continuous.
9. Let $f(x) = |x|$. Find the four derivatives at $x = 0$.
10. Let $f = g$ a.e. (μ), where μ is a complete measure. Show that g is measurable if f is measurable.
11. Let μ be σ-finite measure and υ a σ-finite signed measure and let $\upsilon \ll \mu$; Show that $\frac{d|\upsilon|}{d\mu} = \left|\frac{d\upsilon}{d\mu}\right| [\mu]$.
12. Let μ be a measure and let the measures υ_1, υ_2 be given by $\upsilon_1(E) = \mu(A \cap E), \upsilon_2(E) = \mu(B \cap E)$, where $\mu(A \cap B) = 0$ and $E, A, B \in S$. Show that $\upsilon_1 \perp \upsilon_2$.

Turn over

13. Let f be a finite-valued monotone increasing function defined on (a,b). Show that $g(x) = f(x-)$ is left-continuous and monotone increasing on (a,b).

14. Give an example of a function which is continuous on R but not absolutely continuous.

(14 × 1 = 14 weightage)

Part B

Answer any **seven** *questions.*
Each question carries a weightage of 2.

15. Prove that every interval is measurable.

16. Show that not every measurable set is a Borel set.

17. State and prove Lebesgue's dominated convergence theorem.

18. Show that $\int_0^1 \sin x \log x \, dx = \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n)(2n)!}$.

19. Let f be finite-valued monotone increasing function on $[a,b]$, then prove that f is continuous except on a set of points which is at most countable.

20. If $f \in L(a,b) \wedge \int_a^x f \, dt = 0$ for all $x \in (a,b)$ then prove that $f = 0$ a.e.

21. If μ is a σ finite measure on a ring R, then prove that it has a unique extension to the σ ring S(R).

22. Prove that a union of sets positive with respect to a signed measure υ is a positive set.

23. Let f be absolutely continuous on $[a,b]$, where a and b are finite, then prove that $f \in BV[a,b]$.

24. Let f and g be absolutely continuous on the finite interval $[a,b]$. Show that fg is absolutely continuous on $[a,b]$.

(7 × 2 = 14 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 4.

25. Prove that there exists a non-measurable set.

26. State and prove Fatou's Lemma.

27. State and prove Jordan decomposition theorem.

28. State and prove Riesz Representation theorem for C(I).

(2 × 4 = 8 weightage)