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Name.......................................

Reg. No....................................

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION, JUNE 2020

(CBCSS)

Mathematics

MT 2C 06—ALGEBRA—II

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*

Each question carries 1 weightage.

1. Show that if R is a ring with unity, and N is an ideal of R containing a unit, then N = R.
2. If $\alpha = \sqrt{2} + i$, then determine $\deg(\alpha, Q)$.
3. Find the degree and a basis for the field extension $Q(\sqrt{2}, \sqrt{3})$ over Q.
4. Let E be a finite extension of degree n over a finite field F. Show that if F has q elements, then E has q^n elements.
5. State Isomorphism extension theorem.
6. Show that if $\alpha, \beta \in \overline{F}$ are both separable over a field F, then both $\alpha + \beta$ and $\alpha\beta$ are separable over F.
7. Show that the Galois group of the p^{th} cyclotomic extension of Q for a prime p is cyclic of order $p - 1$.
8. Find $\phi_3(x)$ over Z_2.

($8 \times 1 = 8$ weightage)

Part B

Answer any **two** *questions from each of the following 3 units.*

Each question carries 2 weightage.

UNIT I

9. If R is a ring with unity 1, then show that the map $\phi : Z \to R$ given by $\phi(n) = n.1$ for $n \in z$ is a homomorphism of Z into R.
10. Show that a field F is algebraically closed iff every non-constant polynomial in $F[x]$ factors in $F[x]$ into linear factors.
11. Show that 60° cannot be trisected.

Turn over

UNIT II

12. Find all primitive 10th roots of unity in Z_{11}.

13. Let $\{\sigma_i : i \in I\}$ be a collection of automorphisms of a field E. Show that the set $E_{\{\sigma_i\}}$ of all $a \in E$ left fixed by every σ_i for $i \in I$ forms a subfield of E.

14. Show that $E \leq \overline{F}$ is a splitting field over a field F, then every irreducible polynomial in $F[x]$ having a zero in E splits in E.

UNIT III

15. Describe the group of the polynomial $(x^8 - 1) \in Q[x]$ over Q.

16. Show that the regular n-gon is constructible only if $\phi(n)$ is a power of 2.

17. Verify whether the splitting field of $x^{17} - 5$ over Q has a solvable Galois group.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 *weightage.*

18. (a) Let F be a field and let $f(x)$ be a non-constant polynomial in $F[x]$. Show that there exists an extension field E of F and an $\alpha \in E$ such that $f(\alpha) = 0$.

    (b) Show that a finite extension field E of a field F is an algebraic extension of F.

19. (a) Show that a finite field of p^n elements exists for every prime power p^n.

    (b) Let F be a finite field of characteristic p. Show that the map $\sigma_p : F \to F$ defined by $\sigma_p(a) = a^p$ for $a \in F$ is an automorphism. Further show that $F_{\{\sigma_p\}} = Z_p$. Define perfect field and show that every finite field is perfect.

20. (a) Let K be a finite normal extension of a field F, and let E be an extension of F, where $F \leq E \leq K \leq \overline{F}$. Show that K is a finite normal extension of E, and $G(K|E)$ is precisely the sub-group of $G(K|F)$ consisting of all those automorphisms that leave E fixed.

    (b) Let F be a field of characteristic zero, and let $F \leq E \leq K \leq \overline{F}$, where E is a normal extension of F and K is an extension of F by radicals. Show that $G(E|F)$ is a solvable group.

(2 × 5 = 10 weightage)