C 83066 (Pages : 3) Name..............................

Reg. No..............................

SECOND SEMESTER M.A./M.Sc./M.Com. DEGREE EXAMINATION JUNE 2020

(CBCSS)

Mathematics

MT 2C 10—OPERATIONS RESEARCH

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *the questions.*
Each question has weightage 1.

1. Write an example of a convex function.
2. Write the mathematical form of a general linear programming problem.
3. When is a mathematical programming problem called a convex programming problem ? Explain.
4. When do we say that a transportation problem is balanced ? Explain.
5. Define chain and path in graphs. Prove that path is a chain, but every chain is not a path.
6. For any feasible flow $\{x_i\}, i = 1, 2, ..., m,$ in the graph, prove that the flow x_0 in the return arc is not greater than the capacity of any cut in the graph.
7. What is meant by matrix game ? Describe.
8. Describe the notion of dominance in Game theory.

(8 × 1 = 8 weightage)

Part B

Answer any **two** *questions from each unit.*
Each question has weightage 2.

UNIT I

9. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semidefinite, prove that $f(X)$ is a convex function.

Turn over

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10. Define the dual of a linear programming problem. Prove that dual of the dual is the primal problem.

11. Solve graphically the linear programming problem :

Minimize $z = x_1 + 3x_2$ subject to $x_1 + x_2 \geq 3, -x_1 + x_2 \leq 2, x_1 - 2x_2 \leq 2, x_1 \geq 0, x_2 \geq 0$.

UNIT II

12. If the primal problem is feasible, prove that it has an unbounded optimum if and only if the dual has no feasible solution, and vice versa.

13. Solve the following problem using dual simplex method :

Minimize $x_1 + 2x_2 + 2x_3$ subject to $4x_1 - 5x_2 + 7x_3 \leq 8, 2x_1 - 4x_2 + 2x_3 \geq 2, x_1 - 3x_2 + 2x_3 \leq 2,$ $x_1 \geq 0, x_2 \geq 0, x_3 \geq 0$.

14. Define transportation matrix. Write an example for a transportation matrix.

UNIT III

15. Prove that a tree can have at the most one centre.

16. Describe the branch and bound method in integer programming.

17. Write the formulation of rectangular game as a linear programming problem.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Let $X \in E_n$ and $g_i(X), i = 1, 2, ..., m$ be convex functions in E_n. Let $S \subseteq E_n$ be the set of points satisfying the constraints $g_i(X) \leq 0, \ i = 1, 2, ..., m$. Then prove that S is a convex set.

(b) Use simplex method to maximize : $3x_1 + 2x_2 + 3x_3$ subject to the constraints $x_1 + x_2 + x_3 \leq 9, 2x_1 + 3x_2 + 5x_3 \leq 30, \ 2x_1 - x_2 - x_3 \leq 8, x_1, x_2, x_3 \geq 0$.

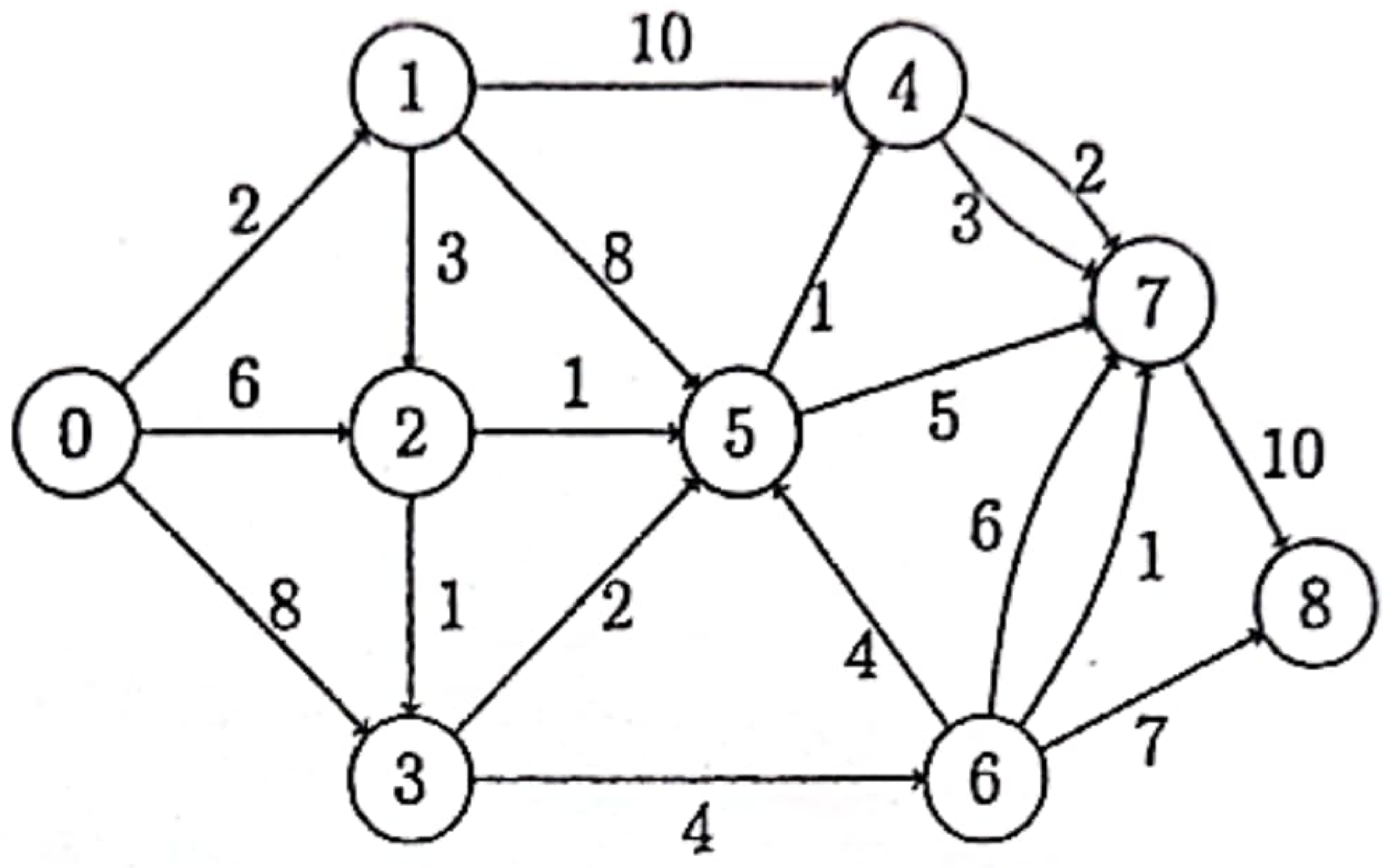
19. (a) Discuss the Caterer problem in operations research.

(b) Solve that the transportation problem for minimum cost with the cost co-efficients, demands and supplies as given in the following table. Obtain three optimal solutions.

20. (a) Solve the following integer linear programming problem :

Maximize $\phi(X) = 3x_1 + 4x_2$; subject to $2x_1 + 4x_2 \le 13$, $-2x_1 + x_2 \le 2$, $2x_1 + 2x_2 \ge 1$, $6x_1 - 4x_2 \le 15$, $x_1, x_2 \ge 0$, x_1 and x_2 are integers.

(b) Find the minimum path from v_0 to v_8 in the following graph in which the number along the directed arcs denote its length.



21. Let $f(X, Y)$ be such that both $\max_X \min_Y f(X, Y)$ and $\min_Y \max_X f(X, Y)$ exist. Then prove that the necessary and sufficient condition for the existence of a saddle point (X_0, Y_0) of $f(X, Y)$ is that $f(X_0, Y_0) = \max_X \min_Y f(X, Y) = \min_Y \max_X f(X, Y)$.

(2 × 5 = 10 weightage)