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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2024

(CBCSS)

Mathematics

MTH2C10—OPERATIONS RESEARCH

(2019 Admission onwards)

e : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1. Give an example of a function which is both convex and concave.
2. Define the term 'cost co-efficients' associated with a linear programming problem.
3. Write the following LP problem in standard form (minimize CX subject to AX = B, X $\geq$ 0.)

   Maximize $2x_1 + 3x_2 + 5x_3$

   subject to $x_1 + x_2 - x_3 \geq -5$

   $-6x_1 + 7x_2 - 9x_3 \leq 4$

   $x_1 + 8x_2 + 4x_3 = 12$

   $x_1 \geq 0, x_2 \geq 0, x_3$ unrestricted in sign.

4. Briefly describe a transportation problem.
5. What is meant by loop in a transportation array.
6. Prove that, for any feasible flow $\{x_i\}, i = 1, 2, ..., m$, in the graph, the flow x_0 in the return arc is not greater than the capacity of any cut in the graph.
7. Describe the effect of introducing new variables on the optimal solution of an LP problem.
8. Define the term 'pay off' associated with games.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **six** *questions, by choosing two questions from each module.*
Each question has weightage 2.

Module I

9. Let $X \in E_n$ and let $f(X) = X'AX$ be a quadratic form. If $f(X)$ is positive semidefinite that $f(X)$ is a convex function.

10. Solve graphically :

Maximize $5x_1 - x_2$

subject to $x_1 + x_2 \geq 2$

$x_1 + 2x_2 \leq 2$

$2x_1 + x_2 \leq 2$

$x_1, x_2 \geq 0.$

11. What is meant by simplex multipliers. Explain with a suitable example.

Module II

12. Prove that if the kth constraint of the primal is an equality then the dual variable y_k is in sign.

13. Briefly describe the dual simplex method.

14. The table shown below gives the quantity of goods available at four origins O_i, $i = 1$ the minimum requirement at three destinations, D_j, $j = 1, 2, 3$, and the cost of trans unit quantity of goods from origins to destinations. The available goods exceed the mi requirement, and the excess can be transported to the destinations, but at minimum c distribution of goods such that the total cost of transportation is minimum.

	D_1	D_2	D_3	
O_1	2	1	3	10
O_2	4	5	7	25
O_3	6	0	9	25
O_4	1	3	5	30
	20	20	15	

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MODULE III

15. Show that the optimal solution of the following problem for $\lambda = 0$ remains optimal for $0 \leq \lambda \leq 2/3$, and find the solution.

Maximize $3x_1 + 6x_2$

subject to : $(1 + 2\lambda)\, x_1 \leq 4$

$3(1 - \lambda)\, x_1 + 2x_2 \leq 18$

$x_1 \geq 0, x_2 \geq 0.$

16. Briefly describe the branch and bound method to solve an integer linear programming problem.

17. Solve the game with the pay-off matrix :

$$\begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}.$$

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

18. (a) Let $f(X)$ be defined in a convex domain $K \subseteq E_n$ and be differentiable. Prove that $f(X)$ is a convex function if and only if :

$$f(X_2) - f(X_1) \geq (X_2 - X_1)' \nabla f(X_1)$$

for all X_1, X_2 in K.

(b) Solve the following problem using simplex method :

Maximize $5x_1 + 3x_2 + x_3$

subject to $2x_1 + x_2 + x_3 = 3$

$-x_1 + 2x_3 = 4$

$x_1, x_2, x_3 \geq 0.$

Turn over

19. (a) Prove that the optimum value of $f(X)$ of the primal, if it exists, is equal to the op

of ϕ (Y) of the dual.

(b) A caterer needs clean table covers every day for six days to meet a contract acc

following schedule.

Days	...	1	2	3	4	5	6
Number of covers	...	50	60	80	70	90	100

The cost of a new cover is Rs. 20 while washing charges are Rs. 1 for return on t

or later, Rs. 2 for return on the third day and Rs. 3 for the next day. Find the m

schedule for the purchase and washing of table covers, assuming that after t

contract the covers are rejected.

20. (a) Find the maximum non-negative flow in the network described below, arc (v_j, v_k) i

as (j, k); v_a is the source and v_b is the sink :

Arc	:	$(a, 1)$	$(a, 2)$	$(1, 2)$	$(1, 3)$	$(1, 4)$	$(2, 4)$	$(3, 2)$	$(3, 4)$	$(4, 3)$	(3.
Capacity	:	8	10	3	4	2	8	3	4	2	1

(b) Solve the following problem by cutting plane method :

Maximize $4x_1 + 5x_2$

subject to $3x_1 + x_2 \geq 2$

$x_1 + 4x_2 \geq 5$

$3x_1 + 2x_2 \geq 7$

x_1, x_2 non-negative integers.

21. (a) Briefly describe what is parametric linear programming.

(b) Prove that, for an $m \times n$ matrix game, both $\max_X \min_Y E(X, Y)$ and $\min_Y \max_X$ E

and are equal.

(2 × 5 = 1