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Name.......................................

Reg. No...................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2024

(CBCSS)

Mathematics

MTH 2C 06—ALGEBRA—II

(2019 Admission onwards)

Time : Three Hours | Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Find a maximal ideal of the ring $\mathbb{Z}_6$ of integers mod 6.

2. Verify whether $\mathbb{Z} \times \{0\}$ is a prime ideal of $\mathbb{Z} \times \mathbb{Z}$.

3. Verify whether $\sqrt{2}$ is algebraic over the field $\mathbb{Q}$ of rationals.

4. Verify whether there exists a field of 10 elements.

5. Find all automorphisms of the field $\mathbb{C}$ of complex numbers leaving all reals fixed.

6. Verify whether $\mathbb{Q}\left(\sqrt[3]{2}\right)$ is a splitting field over $\mathbb{Q}$.

7. Verify whether $y_1^2 + y_1y_2 + y_2^2$ is a symmetric function in y_1, y_2.

8. Describe extension by radicals of a field F.

(8 × 1 = 8 weightage)

Turn over

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Part B

*Answer any **two** questions from each module.*
Each question has weightage 2.

Module I

9. Let R be a ring with unity and of characteristic 2. Show that R contains a subring isom the ring $\mathbb{Z}_2$.

10. Let E be an extension of a field F and $\alpha \in E$ be transcendental over F. Show that the eva homomorphism $\phi_\alpha : F[x] \to E$ is a one to one map.

11. Verify whether $\mathbb{Q}(\sqrt{\pi})$ is an algebraic extension of $\mathbb{Q}(\pi)$.

Module II

12. Let F be a finite field of characteristic p. Show that the number of elements in F is p^n positive integer n.

13. Let $E = \mathbb{Q}(\sqrt{2}, \sqrt{3})$. Find the index $\{E : \mathbb{Q}\}$.

14. Show that every finite extension of a field of characteristic zero is a separable extensio

Module III

15. Let $K = \mathbb{Q}(\sqrt{2}, \sqrt{3})$ and $E = \mathbb{Q}(\sqrt{2})$. Find the subgroup of the Galois group $G(K/\mathbb{Q})$ corr to E in the Galois correspondence.

16. Show that the 8th cyclotomic polynomial $\phi_8(x)$ is $x^4 + 1$.

17. Show that the regular 7-gon is not constructive.

$(6 \times 2 = 12)$

Part C

*Answer any **two** questions.*
Each question has weightage 5.

18. (a) Define maximal ideal in a ring.

    (b) Let R be a commutative ring with unity and M be an ideal of R. Show that M i ideal of R if and only if R/M is a field.

19. Let E be an extension of a field F and $\alpha \in E$ be algebraic over F. Show that there is an irreducible polynomial $p(x) \in F[x]$ such that

    (a) $p(\alpha) = 0$ and

    (b) if $f(x) \in F[x]$ is a nonzero polynomial with $f(\alpha) = 0$ then $p(x)$ divides $f(x)$.

20. (a) Show that for every prime p and every positive integer n there is a finite field of order p^n.

    (b) Let E be a finite extension of the field $\mathbb{Z}_p$. Show that there is an $\alpha \in E$ such that $E = \mathbb{Z}_p(\alpha)$.

21. (a) Define the n^{th} cyclotomic polynomial $\phi_n(x)$ over a field F.

    (b) Prove that :

    (i) $\phi_n(x) \in F[x]$.

    (ii) $\phi_n(x)$ divides $x^n - 1$.

(2 × 5 = 10 weightage)