

C 42789

(Pages : 3)

Name.....

Reg. No.....

**SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY)
EXAMINATION, APRIL 2023**

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A*Answer all questions.**Each question carries 1 weightage.*

1. Define base of a topological space. Illustrate with an example.
2. Prove that a discrete space is second countable iff the underlying set is countable.
3. Let (X, τ) be a topological space and let $A \subset X$. Then prove that $\overline{\overline{A}} = \overline{A}$.
4. Define a local base at a point x in a space X . Give an example. When will you say that a space is first countable?
5. When will you say that a space is connected? Prove that if X is connected then X cannot be written as the disjoint union of two nonempty closed sets.
6. In a Hausdorff space, prove that limits of sequences are unique.
7. State Urysohn's lemma.
8. Prove that every compact Hausdorff space is a T_3 space.

(8 × 1 = 8 weightage)

Turn over

Part B

Answer any **two** questions from each unit.
Each question carries 2 weightage.

UNIT 1

- 9 Prove that the real line with the semi-open interval topology is not second countable.
- 10 Let (X, \mathcal{T}) be a topological space and let $\mathcal{B} \subset \mathcal{T}$. Then prove that \mathcal{B} is a base for \mathcal{T} if $x \in X$ and any open set G containing x , there exist $b \in \mathcal{B}$ such that $x \in b$ and $b \subset G$.
- 11 Let (X, \mathcal{T}) and (Y, \mathcal{U}) be topological spaces and let $f: X \rightarrow Y$ be any function. If f is continuous at $x_0 \in X$, then prove that for every subset A of X , $x_0 \in \overline{A}$ implies $f(x_0) \in \overline{f(A)}$.

UNIT 2

- 12 Prove that the co-countable topology on a set makes it into a Lindeloff space.
- 13 Prove that the product topology is the weak topology determined by the projection functions.
- 14 Prove that every closed, surjective map is a quotient map.

UNIT 3

- 15 Suppose a topological space has the property that for every closed subset A of X , every real valued function on A has a continuous extension to it. Then prove that X is normal.
- 16 Prove that all metric spaces are T_4 .
- 17 Suppose y is an accumulation point of a subspace A of a T_1 space X . Then every neighborhood of y contains infinitely many points of A .

(6 × 2 = 12)

Part C

Answer any **two** from the following four questions.
Each question carries 5 weightage.

18. (a) For a subset A of a topological space X , prove that $\overline{A} = A \cup A'$.
- (b) Define sub-base for a topology. Let (X, \mathcal{T}) be a topological space and S a family of subsets of X . Then S is a sub-base for \mathcal{T} if and only if S generates \mathcal{T} .

19. (a) The topological product of a finite number of connected spaces is connected. Prove the statement.
(b) Prove that every closed and bounded interval is compact.
20. (a) For a topological space X , prove that the following statements are equivalent :
(i) X is locally connected.
(ii) Components of open subsets of X are open in X .
(iii) X has a base consisting of connected subsets.
(iv) For every $x \in X$ and every neighbourhood N of x , there exist a connected open neighbourhood M of x such that $M \subset N$.
- (b) Prove that every quotient space of a locally connected space is locally connected.
21. (a) Prove that the axioms T_0, T_1, T_2, T_3 and T_4 form a hierarchy of progressively stronger conditions.
(b) Give an example of :
(i) a T_0 space which is not T_1 .
(ii) a T_1 space which is not T_2 .
(iii) a T_2 space which is not T_3 .

(2 × 5 = 10 weightage)