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Name.....................................

Reg. No.....................................

ECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

: Three Hours

Maximum : 30 Weightage

General Instructions

Covid Instructions are not applicable for Pvt/SDE students

. *In cases where choices are provided, students can attend* **all** *questions in each section.*

. *The minimum number of questions to be attended from the Section / Part shall remain the same.*

. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*

4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries weightage 1.

1 Define neighbourhood of a point in a topological space. Prove that a subset of a topological space is open iff it is a neighbourhood of each of its points.

2 Explain Sierpinski space. Prove that the Sierpinski space is not obtainable from a pseudometric.

3 Let $(X, \mathfrak{T})$ be a topological space and let $A \subset X$. Then prove that $\overline{\overline{A}} = \overline{A}$.

4 Prove that the property of being a discrete space is divisible.

5 Prove that compactness is preserved under continuous functions.

6 In a Hausdorff space, prove that limits of sequences are unique.

Turn over

7

7 State Urysohn's lemma.

8 Let X be a topological space and (Y, d) be a metric space. Let (f_n) be a sequence of f... X to Y converging uniformly to $f : X \to Y$. Then prove that if each f_n is continuous, ...

(8 × 1 = ...

Part B

Answer any **two** *questions from each unit.*
Each question carries weightage 2.

UNIT I

9 Let $(X, \mathfrak{T})$ be a topological space and S be a family of subsets of X. Then prove that S i... of X iff S generates $\mathfrak{T}$.

10 For a subset A of a space X, prove that :

$\bar{A} = \{y \in X :$ every neighbourhood of y meets A non-vacuously$\}$.

11 Let $(X, \mathfrak{T})$ and $(Y, \mathfrak{U})$ be topological spaces and let $f : X \to Y$ be any function. If f is ... at $x_0 \in X$, then prove that for every subset A of X, $x_0 \in \bar{A}$ imples $f(x_0) \in \overline{f(A)}$.

UNIT II

12 Let $\mathcal{C}$ be a collection of connected subsets of a space X such that no two members of $\mathcal{C}$ ar... separated. Then prove that union of all elements of $\mathcal{C}$ is connected.

13 Prove that the product topology is the weak topology determined by the projection fun...

14 Prove that every quotient space of a locally connected space is locally connected.

UNIT III

15 Prove that a space X is a T_1 space iff every finite subset of X is closed.

16 Prove that all metric spaces are T_4.

17 Prove that there can be no continuous one-to-one map from the unit circle S^1 into the re...

(6 × 2 = 12 we...

Part C

Answer any **two** *questions.*
Each question carries weightage 5.

18. (a) For a subset A of a topological space X, prove that $\bar{A} = A \cup A'$.

(b) Let $\{(X_i, \mathfrak{T}_i) : i = 1, 2, \ldots n\}$ be a collection of topological spaces and $(X, \mathfrak{T})$ their topological product. Then prove that each projection π_i is continuous. Moreover, if Z is any space, then prove that a function $f : Z \to X$ is continuous iff $\pi_i \circ f : Z \to X_i$ is continuous for all $i = 1, 2, \ldots n$.

19. (a) Let (X, d) be a compact metric space and let $\mathfrak{U}$ be an open cover of X. Then prove that there exist a positive real number r such that for any $x \in X$ there exist $V \in \mathfrak{U}$ such that $B(x, r) \subset V$.

(b) Prove that every second countable space is first countable. Is the converse true ? Justify your answer.

20. (a) In a topological space X, prove the following :

(i) Components are closed sets.

(ii) Any two distinct components are mutually disjoint.

(iii) Every non-empty connected subset is contained in a unique component.

(iv) Every space is the disjoint union of its components.

(b) Prove that every open subset of the real line in the usual topology can be expressed as the union of mutually disjoint open intervals.

21 State and prove the Tietze extension theorem.

($2 \times 5 = 10$ weightage)