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Name..

Reg. No.......................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

(CBCSS)

Mathematics

MTH 2C 10—OPERATIONS RESEARCH

(2019 Admission onwards)

e : Three Hours | Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Prove that $f(x) = x_1^2 + x_2^2 + 4x_3^2 - 2x_1x_2$ is a convex function.

2. If $f(X)$ is minimum at more than one of the vertices of S_F, then show that it is minimum at all those points which are the convex linear combinations of these vertices.

3. Write the following LP in the standard form :

Minimize $f = x_1 + x_2 - x_3$

subject to $2x_1 + 3x_2 + x_3 \geq 1$

$x_1 + 2x_2 - x_3 \leq 2$

$3x_1 + 2x_2 - x_3 = 5$

$x_1 \geq 0, x_2 \geq 0, x_3$ is unrestricted.

Turn over

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4. Show that the dual of the dual is the primal.

5. State the complementary slackness conditions.

6. Find the optimal strategies and the value of the game $\begin{pmatrix} -4 & 6 & 3 \\ -3 & -3 & 4 \\ 2 & -3 & 4 \end{pmatrix}$.

7. Solve the game $\begin{pmatrix} 1 & 7 \\ 6 & 2 \end{pmatrix}$.

8. Briefly explain the deletion of a variable while determining a new optimal solution fr solution already obtained.

(8 × 1

Part B

Answer any **two** *questions from each of the following* 3 *units.*
Each question carries 2 *weightage.*

Unit I

9. Using graphical method solve the following linear programming problem.

Maximize $f(x) = 3x_1 + 5x_2$

subject to $x_1 + 2x_2 \leq 20$

$x_1 + x_2 \leq 15$

$x_2 \leq 6$

$x_1 \geq 0, x_2 \geq 0.$

10. Show that a vertex of the set S_F of feasible solutions is a basic feasible solution.

11. Characterize all convex functions f (X) which are defined in a convex domain I differentiable.

Unit II

12. Explain by an example the method of finding a basic feasible solution for a transpo

13. Show that the transportation problem has a triangular basis.

14. Explain the dual simplex method using the following example :

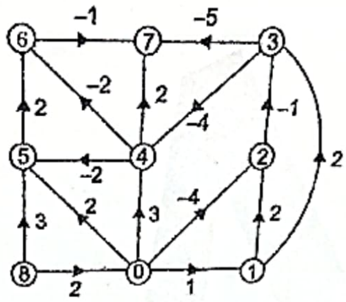
Minimize $f = 3x_1 + 5x_2 + 2x_3$

subject to
$$-x_1 + 2x_2 + 2x_3 \geq 3$$
$$x_1 + 2x_2 + x_3 \geq 2$$
$$-2x_1 - x_2 + 2x_3 \geq -4$$
$$x_1, x_2, x_3 \geq 0.$$

Unit III

15. State and prove the fundamental theorem of rectangular games.

16. Find the minimum path from V_0 to V_7 in the following graph :



17. Minimize $f = 3x_4 + 4x_5 + 5x_6$

subject to
$$2x_1 - 2x_4 - 4x_5 + 2x_6 = 3$$
$$2x_2 + 4x_4 + 2x_5 - 2x_6 = 5$$
$$x_3 - x_4 + x_5 + x_6 = 4$$
$$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$$
$$x_1, x_2 \text{ integers.}$$

($6 \times 2 = 12$ weightage)

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. Explain the simplex method by solving the following linear programming problem :

Maximize $f(x) = 4x_1 + 5x_2$

subject to
$$x_1 - 2x_2 \leq 2$$
$$2x_1 + x_2 \leq 6$$
$$x_1 + 2x_2 \leq 5$$
$$-x_1 + x_2 \leq 2$$
$$x_1 \geq 0, x_2 \geq 0.$$

Turn over

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. Solve the following transportation problem :

	D_1	D_2	D_3	
O_1	2	1	3	10
O_2	4	5	7	25
O_3	6	0	9	25
O_4	1	3	5	30
	20	20	15	

20. Write an algorithm to find a minimum spanning tree and prove it.

21. Explain the cutting plane method with an example.

(2 × 5 = 10