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Name.......................................

Reg. No..................................

SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2022

April 2021 Session for SDE/Private Students

(CBCSS)

Mathematics

MTH 2C 09—ODE AND CALCULUS OF VARIATIONS

(2019 Admission onwards)

ime : Three Hours Maximum : 30 Weightage

General Instructions

Covid Instructions are not applicable for SDE/Private Students

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Section A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Find the value of a_0 in the Fourier series expansion of $f(x) = x, -\pi \le x \le \pi$.
2. Write the Legendre's differential Equation.
3. State Dirichlet's theorem.
4. Find co-efficient a_n in the cosine series expansion for the function $f(x) = \sin x,\ 0 \le x \le \pi$.
5. Find the general solution of $y'' + 9y = 0$.

Turn over

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6. Find the Fourier series expansion of the function $f(x) = \cos^2 x$.

7. Write the Rodrigue's formula for the Legendre polynomials.

8. Define Lipschitz Continuity of a function. Give an example.

(8 × 1 = 8 weigh

Section B

Answer any **two** *questions from each of the following three units.*
Each question carries 2 weightage.

Unit I

9. Solve $xy'' - (x+2)y' + 2y = 0$.

10. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{(n+1)}{(n+2)(n+3)} x^n$.

11. Show that $J_n'(x) = J_{n-1}(x) - \frac{n}{x} J_n(x)$.

Unit II

12. Show that $\Gamma(p+1) = p\Gamma(p)$.

13. Show that the functions e^{3t} and e^{2t} are linearly independent.

14. Find the second Picard approximation for the solution of the equation $\frac{du}{dx} = u^2 + x$ giv

$u = 0$ when $x = 0$.

Unit III

15. Test for linear independence of the functions $\sin x, \sin 3x, \sin^3 x$.

16. Express $x^4 - 3x^2 + x$ in terms of Legendre polynomials.

17. Prove that $P_n(x) = {}_2F_1\left(-n, n+1, 1, \frac{1-x}{2}\right)$.

(6 × 2 = 12 we

Section C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. Solve in series $\left(1-x^2\right)y''-xy'+4y=0.$

19. If $m>(n-1)$ and n is a positive integer, show that

$$\int_0^1 x^m \, P_n(x)\, dx = \frac{m(m-1)(m-2)\ldots\ldots(m-n/2)}{(m+n+1)(m+n-1)\ldots.(m-n+3)}.$$

20. Show that $J_n(-x)=(-1)^n J_n(x)$.

21. Extremize the functional $V\left[y(x)\right]=\int_{x_0}^{x_1}\frac{1+y^2}{(y')^2}\,dx.$

(2 × 5 = 10 weightage)