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SECOND SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, APRIL 2024

(CBCSS)

Mathematics

MTH 2C 08—TOPOLOGY

(2019 Admission onwards)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Let d be a metric on the reals $\mathbb{R}$ given by

$$d(x, y) = \begin{cases} 0 \text{ if } x = y \\ 1 \text{ otherwise} \end{cases}.$$

Find all points in the open ball of radius 1 centered at 0.

2. Let τ be the cofinite topology on $\mathbb{R}$. Verify whether $A = \{x : x > 0\}$ is in τ.

3. Let A be an open set in X and B be open in Y. Verify whether A × B is open in the product space X × Y.

4. Let X be the set of reals and Y be the space $\mathbb{R}$ with usual topology. Let $f : X \to Y$ be defined by

$$f(x) = \begin{cases} 0 \text{ if } x \text{ is rational} \\ 1 \text{ otherwise} \end{cases}.$$

Find a topology on X other than the discrete topology such that f is continuous.

5. Give an example of a first countable space.

6. Let $X = \mathbb{R}$ with discrete topology. Find the connected components of X.

Turn over

7. Let $X = \{1, 2, 3, 4\}$ and $\tau = \{X, \phi, \{1, 2\}, \{1, 2, 3\}\}$. Verify whether X is a T_1-space.

8. Let X be a metric space and a, b be distinct points in X. Find disjoint open neighbourhoo
b in X.

(8 × 1 = 8

Part B

Answer any **two** *questions from each module.*
Each question has weightage 2.

Module I

9. Show that the set

$$\mathcal{B} = \{A \subseteq \mathbb{R} : \mathbb{R} \sim A \text{ is singleton}\}$$

is a subbase for the cofinite topology on $\mathbb{R}$.

10. Let A be a dense subset of a topological space X and G be an open set in X. Show th
non-empty.

11. Let X, Y be topological spaces and X × Y be the product space. Show that th
$p_1 : X \times Y \to X$ is continuous.

Module II

12. Let $f_i : X \to Y_i$ be a family of functions from a set X to topological spaces Y_i. Show that
on X with subbase

$$S = \{f_i^{-1}(V_i) : V_i \text{ is open in } Y_i\}$$

is the smallest toplogy on X making each f_i continuous.

13. Show that every second countable space is first countable.

14. Show that the closed interval [0, 1] is a connected subspace of the real line.

Module III

15. Let X be a T_1-space. Show that for each $x \in X$ the singleton set $\{x\}$ is a closed subspace of X.

16. Show that every compact subset of a Hausdorff space is closed.

17. Let X be a regular space and Y be a subspace of X. Show that Y is regular.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Define base for a topology.

    (b) Let $\mathcal{B}$ be a subset of a topology τ on X. Show that $\mathcal{B}$ is a base for τ if and only if for any $x \in X$ and any open set G containing x there is a $B \in \mathcal{B}$ such that $x \in \mathcal{B}$ and $B \subseteq G$.

19. (a) Define the closure $\overline{A}$ of a subset A of a topological space X.

    (b) Show that

    i) if A is closed then $\overline{A} = A$.

    ii) if A, B are subsets of X then $\overline{A \cup B} = \overline{A} \cup \overline{B}$.

20. (a) Define path connected space.

    (b) Show that every path connected space is connected.

    (c) Give an example of a connected space which is not path connected.

21. (a) Let X be a Hausdorff space and $x \in X$. Let F be a compact subset of X not containing x. Show that there exist disjoint open sets U, V such that $x \in U$ and $F \subseteq V$.

    (b) Show that every compact Hausdorff space is a T_3-space.

(2 × 5 = 10 weightage)