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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2024

(CBCSS)

Mathematics

MTH 1C 02—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question carries 1 weightage.

1. Define Vector Space.
2. Find two linear operators T and U on $\mathbb{R}^2$ such that $TU = 0$ but $UT \neq 0$.
3. Find the dimension of the null space of a nonzero linear functional on a finite dimensional vector space V.
4. Let F be a field and let $\alpha = (x_1, \ldots\ldots, x_n)$ be a vector in F^n. Find $[\alpha]_B$, where B is the standard ordered basis for F^n.
5. Show that similar matrices have the same characteristic polynomial.
6. Let V be a vector space and E be a projection of V. Prove that a vector β is in range of E if and only if $E\beta = \beta$.
7. Let V be a vector space and (/) an inner product on V. Show that $(0/\beta) = 0$ for all $\beta \in V$.
8. Verify that the standard inner product on F^n is an inner product.

(8 × 1 = 8 weightage)

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Part B (Paragraph Type Questions)

Answer any **two** *questions, choosing two questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Show that a nonempty subset W of V is a subspace of V if and only if for each pair of vecto[r] in W and each scalar c in F the vector $c\alpha + \beta$ is again in W.

10. Let V and W be two vector spaces over the field F . Let T and U be linear transformations f into W and c is any element of F. Define $(T+U)$ by $(T+U)(\alpha) = T(\alpha) + U(\alpha)$ and $(cT)(\alpha) = cT(\alpha)$ for $\alpha \in V$. Show that the set of all linear transformations from V into W , to[g] with the addition and scalar multiplication defined above, is a vector space over the field F

11. Let V, W and Z be vector spaces over the field F. Let T be a linear transformation from V i[nto W] and U be a linear transformation from W into Z. Then show that the composition function defined by $(UT)(\alpha) = U(T(\alpha))$ is a linear transformation from V into Z.

MODULE II

12. Let V be the space of all polynomial functions from $\mathbb{R}$ into $\mathbb{R}$ of the form $f(x) = c_0 + c_1x + c_2x^2 +$ Let B be the ordered basis for V consisting of the four functions f_1, f_2, f_3, f_4, defined by $f_j(x) =$ Find $[D]_B$, where D is the differentiation operator of V into V.

13. Let V be a finite dimensional vector space over the field F and let $B = \{\alpha_1, \ldots, \alpha$ $B' = \{\alpha_1', \ldots, \alpha_n'\}$ be ordered bases for V. Suppose T is a linear operator on V. If $P = [P_1, \ldots$ the $n \times n$ matrix with columns $P_j = [\alpha_j']_B$, then prove that $[T]_{B'} = P^{-1}[T]_B P$.

14. Let A be the (real) 3×3 matrix $\begin{bmatrix} 3 & 1 & -1 \\ 2 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$. Find the characteristic values and charact vectors of A.

MODULE III

15. For $\alpha = (x_1, x_2)$ and $\beta = (y_1, y_2) \in \mathbb{R}^2$, define $(\alpha/\beta) = x_1y_1 - x_2y_1 - x_1y_2 + 4x_2y_2$. Verify that (/) is an inner product

16. If $V = W_1 \oplus \ldots \oplus W_k$, then show that there exist k linear operators $E_1, \ldots, E_k$ on V such that :

(i) each E_i is a projection ;

(ii) $E_iE_j = 0$ if $i \neq j$;

(iii) $I = E_1 + \ldots + E_k$; and

(iv) the range of E_i is W_i .

17. Let W be a finite-dimensional subspace of an inner product space V and let E be the orthogonal projection of V on W. Then prove that E is an idempotent linear transformation of V onto W, $W^{\perp}$ is the null space of E, and $V = W \oplus W^{\perp}$.

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. If W_1 and W_2 are finite dimensional subspace of a vector space V, then prove that $W_1 + W_2$ is finite dimensional and $\dim W_1 + \dim W_2 = \dim (W_1 \cap W_2) + \dim (W_1 + W_2)$.

19. Let V and W be finite-dimensional vector spaces over the same field F such that dim V = dim W. If T is a linear transformation from V into W, then prove that the following are equivalent :

(i) T is invertible

(ii) T is non-singular

(iii) T is onto ; i.e., the range of T is W.

(iv) If $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ is a basis for V, then $\{T\alpha_1, T\alpha_2, \ldots, T\alpha_n\}$ is a basis for W.

(v) There is some basis $\{\alpha_1, \alpha_2, \ldots, \alpha_n\}$ for V such that $\{T\alpha_1, T\alpha_2, \ldots, T\alpha_n\}$ is a basis for W.

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20. If f is a non-zero linear functional on the vector space V, then prove that the null space of hyperspace in V. Conversely, show that every hyperspace in V is the null space of a (not ur non-zero linear functional on V.

21. State and prove Gram-Schmidt Orthogonalization process. Consider vectors $\beta_1 = (3, 0, 4), \beta_2 = (-1, 0, 7), \beta_3 = (2, 9, 11)$ in $\mathbb{R}^3$ with standard inner product. Apply (Schmidt Orthogonalization process to $\beta_1, \beta_2, \beta_3$, and obtain an orthonormal basis for $\mathbb{R}^3$.

(2 × 5 = 10 weig]