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Name.....

Reg. No.....

FIRST SEMESTER M.Sc. (CBCSS) REGULAR/SUPPLEMENTARY DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admission onwards)

Time : Three Hours

Maximum Weightage : 30

Part A

Answer **all** *questions.*
Each question carries 1 *weightage.*

1. Prove that $d(n)$ is odd if and only if n is a square.
2. Show that $\phi(n) > \frac{n}{6}, \forall n$ with atmost eight distinct prime factors.
3. Prove that the Mobious function is multiplicative but not completely multiplicative.
4. Define Chebyshev's function $\psi(x)$ and $\vartheta(x)$ and show that $\psi(x) = \sum_{m \le \log_2 x} \vartheta(x)$.
5. State Shapiro's Tauberian theorem.
6. If $x \ge 1$ show that $\sum_{n \le x} n^{\alpha} = \frac{x^{\alpha+1}}{\alpha+1} + O(x^{\alpha})$, if $\alpha \ge 0$.
7. Determine whether 888 is a quadratic residue or nonresidue of the prime 1999.
8. Briefly describe about digraph transformation.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions by choosing* **two** *questions from each unit.*
Each question carries 2 *weightage.*

Unit I

9. Prove that $\phi(n)$ is even for $n \ge 3$. Moreover, if n has r distinct odd prime factors, then $2^r / \phi(n)$.

Turn over

10

10. If f and g are multiplicative, then show that their Dirichlet product $f * g$ is multipilcative.

11. For all $x \geq 1$ prove that $\left|\sum_{n \leq x} \frac{\mu(n)}{n}\right| \leq 1$ with equality holding if $x < 2$.

Unit II

12. State and prove Abel's identity.

13. Let p_n denote the nth prime. Show that the following relations are logically equiva

(i) $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1.$

(ii) $\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1.$

(iii) $\lim_{x \to \infty} \frac{P_n}{n \log n} = 1.$

14. For $n \geq 1$, show that the nth prime p_n satisfies the inequa

$$\frac{1}{6} n \log n < p_n < 12\left(n \log n + n \log \frac{12}{e}\right).$$

Unit III

15. If P is an odd positive integer, show that $(-1/P) = (1)^{\frac{p-1}{2}}$ where $(-1|P)$ denote the symbol.

16. State and prove Quadratic Reciprocity Law.

17. Describe briefly about RSA cryptosystems.

(6 × 2 = 12 weig

Part C

Answer any **two** *questions.*
Each question carries 5 weightage.

18. If $n \geq 1$, show that $\phi(n) = \sum_{d|n} \mu(d) \frac{n}{d}$.

19. For all $x \geq 1$ show that $\sum_{n \leq x} \frac{\wedge(n)}{n} = \log x + O(1)$.

20. State and prove Gauss' lemma.

21. Explain Affine enciphering transformations. In the 27 letter alphabet (with blank = 26), use the affine enciphering transformation within key $a = 13$, $b = 9$ to encipher the message "HELP ME".

(2 × 5 = 10 weightage)

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