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Name.............................

Reg. No.............................

FIRST SEMESTER M.Sc. (CBCSS) [REGULAR/SUPPLEMENTARY] DEGREE EXAMINATION, NOVEMBER 2022

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

Time : Three Hours Maximum Weightage : 30

Part A

Answer **all** *questions.*
Each question carries a weightage 1.

1. Is the relation $a\,|\,b$ meaning 'a divides b' defined on the set of non-negative integers a total ordering ? Justify.

2. Let $(X, \leq)$ be a partially ordered set and let $x \in X$. Let $A = \{z \in X : x < z\}$. Prove that $y \in X$ covers x if and only if y is a minimal element of A.

3. State and prove laws of tautology.

4. Find the complement of the following graph :

5. Prove that the sum of degrees of all the vertices of a graph is equal to twice the number of edges.

6. Draw any two non-isomorphic trees on 4 vertices.

7. Find a grammar that generates $L = \{a^n b^{n+1} : n \geq 0\}$.

8. Prove that $(L_1 L_2)^R = L_2^R L_1^R$.

($8 \times 1 = 8$ weightage)

Turn over

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Part B

Answer any **six** *questions, choosing any* **two** *questions from each unit.*
Each question carries a weightage 2.

Unit I

9. If R is a partial order on a set X then prove that R - ΔX is a strict partial order on
10. Let X be a finite set and let ≤ be a partial order on X. Let R be the binary relation covers x. Prove that ≤ is the smallest order relation on X containing R.
11. Using residue class modulo 8, prove that no integer of the form $8n + 7$, where integer, can be expressed as a sum of three perfect squares.

Unit II

12. Prove that a graph G is bipartite if and only if it does not contain any odd cycle.
13. Prove that an edge $e = xy$ of a connected graph G is a cut edge of G if and only if to no cycle of G.
14. Prove that a tree T on n vertices contains $n - 1$ edges. Is the converse true ? Justi

Unit III

15. Let the grammar $G = (\{S\}, \{a, b\}, S, P)$, with P given by $S \rightarrow aSb$ and $S \rightarrow \lambda$ and grammar $G_1 = (\{A, S\}, \{a, b\}, S, P_1)$ with P_1 given by $S \rightarrow aAb \mid \lambda$ and $A \rightarrow aAb \mid \lambda$. Pr G and G_1 are equivalent.
16. Find a grammar that generates the following languages :
    (i) $L_1 = \left\{a^n b^m ; n \geq 0, m > n\right\}$.
    (ii) $L_2 = \left\{a^n b^{2n} ; n \geq 0\right\}$.
17. Show that the language $L = \left\{awa : w \in \{a, b\}^*\right\}$ is regular.

(6 × 2 = 12 we

Part C

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Let (X, ≤) be a poset and A a non-empty finite subset of X :
    (i) Prove that A has at least one maximal element.
    (ii) Prove also that A has a maximum element if and only if it has a unique element.
    (iii) Give an example of a set which is bounded above but do not have a maximum

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19. Prove that for a nontrivial connected graph G, the following statements are equivalent :

    (i) G is Eulerian.

    (ii) The degree of each vertex of G is an even positive integer.

    (iii) G is an edge-disjoint union of cycles.

20. Prove that K_5 and $K_{3,3}$ are non-planar graphs.

21. Let L be the language accepted by a nondeterministic finite accepter $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Prove that there exists a deterministic finite accepter $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$.

(2 × 5 = 10 weightage)