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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 03—REAL ANALYSIS—I

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer **all** *questions.*

Each question carries a weightage 1.

1. Prove that a finite point set has no limit points.

2. If X is a metric space and $E \subset X$, then prove that $E = \bar{E}$ if and only if E is closed.

3. Discuss the continuity/discontinuity behaviour of the function f defined by

$$f(x) = \begin{cases} 1 & (x \text{ rational}) \\ 0 & (x \text{ irrational}) \end{cases}.$$

4. Suppose f and g are defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$. Then prove that $f + g$ is differentiable at x.

5. If f is a real continuous function on $[a, b]$ which is differentiable in (a, b), then prove that there is a point $x \in (a, b)$ at which

$$f(b) - f(a) = (b - a) f'(x).$$

6. Define upper Riemann integral of a function.

Turn over

7. For $m = 1, 2, 3, \ldots\ldots$, $n = 1, 2, 3, \ldots\ldots$, let

$$S_{m,n} = \frac{m}{m+n}.$$

Then find $\lim_{n \to \infty} \lim_{m \to \infty} S_{m,n}$ and $\lim_{m \to \infty} \lim_{n \to \infty} S_{m,n}$.

8. Describe the test for uniform convergence of sequence of functions, due to Weierstrass.

(8 × 1 = 8 wei

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

Module I

9. If E is an infinite subset of a compact set K, then prove that E has a limit point in K.

10. Prove that subset E of the real line $\mathbb{R}^1$ is connected if it has the following property:

If $x \in E$, $y \in E$, and $x < z < y$, then $z \in E$.

11. Prove that a mapping f of a metric space X into a metric space Y is continuous on X impli is open in X for every open set V in Y.

Module II

12. Suppose f and g defined on $[a, b]$ and are differentiable at a point $x \in [a, b]$. Then pro differentiable at x.

13. Suppose f is differentiable in (a, b). If $f'(x) \geq 0$ for all $x \in (a, b)$, then prove that f is m increasing.

14. If $f_1(x) \leq f_2(x)$ on $[a, b]$, then prove that

$$\int_a^b f_1 d\alpha \leq \int_a^b f_2 d\alpha.$$

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MODULE III

15. If γ' is continuous on $[a, b]$, then prove that γ is rectifiable, and

$$\Lambda(\gamma) = \int_a^b |\gamma'(t)|\, dt.$$

16. Suppose $\{f_n\}$ is a sequence of continuous functions on E, and if $f_n \to f$ uniformly on E, then prove that f is continuous on E.

17. Prove that the sequence of functions $\{f_n\}$, defined on E, converges uniformly on E implies for every $\varepsilon > 0$ there exists an integer N such that $m \geq N, n \geq N, x \in E$ implies

$|f_n(x) - f_m(x)| \leq \varepsilon.$

(6 × 2 = 12 weightage)

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

18. Let A be a the set of all sequences whose elements are the digits 0 and 1. Then prove that the set A is uncountable.

19. Suppose f and g are real and differentiable in (a, b), and $g'(x) \neq 0$ for all $x \in (a, b)$, where $-\infty \leq a < b \leq +\infty$. Suppose

$$\frac{f'(x)}{g'(x)} \to A \text{ as } x \to a.$$

If

$$f(x) \to 0 \text{ and } g(x) \to 0 \text{ as } x \to a,$$

or if

$$g(x) \to +\infty \text{ as } x \to a,$$

Turn over

then prove that

$$\frac{f(x)}{g(x)} \to A \text{ as } x \to a.$$

20. Prove that $f \in \mathfrak{R}(\alpha)$ on $[a, b]$ if and only if for every $\varepsilon > 0$ there exists a partition P such th

$$U(P, f, \alpha) - L(P, f, \alpha) < \varepsilon.$$

21. Suppose K is compact, and

(a) $\{f_n\}$ is a sequence of continuous functions on K,

(b) $\{f_n\}$ converges pointwise to a continuous function f on K,

(c) $f_n(x) \geq f_{n+1}(x)$ for all $x \in K$, $n = 1, 2, 3, \ldots$.

Then prove that $f_n \to f$ uniformly on K.

(2 × 5 = 10 we