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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

Time : Three Hours   Maximum : 30 Weightage

Part A (Short Answer Type Questions)

Answer all questions.
Each question carries a weightage 1.

1. Define strict partial order. Give an example.

2. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that for all elements x and y of X,

   $x + x \cdot y = x.$

3. Give an example of a Boolean function.

4. Define complete graph.

5. Prove that for any simple graph G, $Aut(G) = Aut(G^c)$.

6. If G is a simple planar graph with at least three vertices, then prove that $m \le 3n - 6$.

7. Give an example of a dfa.

8. Define language accepted by an nfa.

              $(8 \times 1 = 8$ weightage)

Turn over

Part B (Paragraph Type Questions)

Answer any **two** *questions from each module.*
Each question carries a weightage 2.

MODULE I

9. Define a chain in a poset. Prove that the intersection of two chains is a chain.

10. Write the following Boolean function in their disjunctive normal form.

$$f\left(x_1, x_2, x_3\right) = \left(x_1 + x_2'\right) x_3' + x_2 x_1' \left(x_2 + x_1' x_3'\right).$$

11. State and prove law of uniqueness of complements in a Boolean algebra.

MODULE II

12. Prove that the number of edges of a simple graph of order n having ω components can

$$\frac{(n-\omega)(n-\omega+1)}{2}.$$

13. Prove that a connected graph G is a tree if and only if every edge of G is a cut edge of

14. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex only if there exist vertices u and w of G distinct from v such that v is in every u-w path

MODULE III

15. Convert the nfa in the following figure into an equivalent deterministic machine.
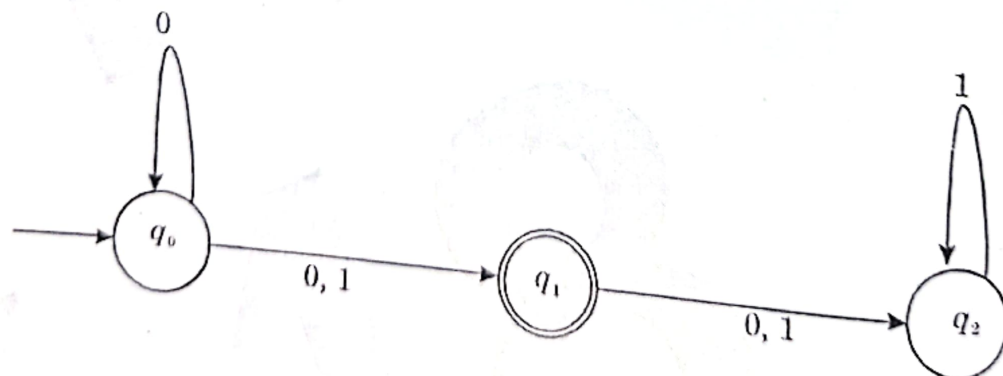
16. Find a deterministic finite accepter that recognizes the set of all strings on $\sum = \{a, b\}$ with the prefix ab.

15

17. Show that the language

$$L = \left\{ awa : w \in \{a, b\}^* \right\}$$

is regular.

Part C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries a weightage 5.

8. Let $(X, +, \cdot, ')$ be a Boolean algebra. Then prove that the corresponding lattice $(X, \leq)$ is complemented and distributive. Conversely prove that if $(X, \leq)$ is bounded, complemented and distributive lattice then there exists a Boolean algebra structure on X, $(X, +, \cdot, ')$ such that the partial order relation defined by this structure coincides with the given relation $\leq$.

9. Prove that a graph is bipartite if and only if it contains no odd cycles.

). Prove that the set $Aut(G)$ of all automorphisms of a simple graph G is a group with respect to the composition ∘ of mappings as the group operation.

.. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite accepter, and let G_M be its associated transition graph. Then prove that for every $q_i, q_j \in Q$, and $w \in \sum^+$, $\delta^* (q_j, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j

(2 × 5 = 10 weightage)