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Name.......................................

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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA—I

(2019 Admission onwards)

Time : Three Hours

Maximum : 30 Weightage

Section A (Short Answer Type Questions)

Answer **all** *questions.*
Each question carries a weightage 1.

1. Find the order of $(2, 6)$ in the group $\mathbb{Z}_4 \times \mathbb{Z}_{12}$.
2. List all abelian groups up to isomorphism of order 16.
3. Let X be a G-set. Prove that G_x is a subgroup of G for each $x \in X$.
4. Let $\phi : \mathbb{Z}_{12} \to \mathbb{Z}_3$ be the homomorphism, where $\phi(1) = 2$. Find Ker ϕ.
5. Find isomorphic refinements of $\{0\} < 10\,\mathbb{Z} < \mathbb{Z}$ and $\{0\} < 25\,\mathbb{Z} < \mathbb{Z}$.
6. Find the number of Sylow 3-Subgroups of a group of order 54.
7. Give a presentation of $\mathbb{Z}_4$ involving three generators.
8. Find the multiplicative inverse of $1 + i + 2j + k$ in the ring of quaternions.

(8 × 1 = 8 weightage)

Turn over

Section B (Paragraph Type Questions)

Answer any **two** *questions from each module.*

Each question carries a weightage 2.

MODULE I

9. If H is a normal subgroup of a group G, then show that the left coset multiplication is well d by the equation $(a\text{H})(b\text{H}) = ab\text{H}$.

10. List all elements of $\mathbb{Z}_2 \times \mathbb{Z}_4$. Is this group cyclic. Justify.

11. Prove that factor group of a cyclic group is cyclic.

MODULE II

12. Find all composition series of $\mathbb{Z}_{48}$.

13. Prove that every group of prime power order is solvable.

14. Prove that every group is a homomorphic image of a free group.

MODULE III

15. Let $\phi_\pi : \mathbb{Q}[x] + \mathbb{R}$ be the evaluation homomorphism with $\phi_\pi(x) = \pi$. Find the kernel of

16. Factorize the polynomial $f(x) = 2x^3 + 3x^2 - 7x - 5$ into linear factors in $\mathbb{Z}_{11}[x]$.

17. Prove that $\mathbb{Z}/n\mathbb{Z} \approx \mathbb{Z}_n$.

(6 × 2 = 12 w

Section C (Essay Type Questions)

Answer any **two** *questions.*

Each question carries a weightage 5.

18. a) Define decomposable group. Prove that the finite indecomposable abelian groups a the cyclic groups with order a power of a prime.

b) If X is a G-set, then prove that the function $\sigma_g : \text{X} \to \text{X}$ defined by $\sigma_g(x) = gx$ is a pe on X. Also show that the map $\phi : \text{G} \to \text{S}_\text{X}$ defined by $\phi(g) = \sigma_g$ is a homomorphis

9. a) State and prove First Sylow Theorem.

b) Prove that no group of order 15 is simple

0. a) Let H and K be normal subgroups of a group G with $K \leq H$. Show that $G/H \simeq {}^{G/K}\!/_{H/K}$.

b) Find the ascending central series of the group S_3.

. a) State and prove Eisenstein Criterion for irreducibility of a polynomial.

b) Show that the polynomial $x^5 + 6x^3 + 4x + 10$ is irreducible over $\mathbb{Q}$.

(2 × 5 = 10 weightage)