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Name.....

Reg. No.....

#### FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2023

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA—I

(2019 Admission onwards)

Time: Three Hours

Maximum: 30 Weightage

#### Section A (Short Answer Type Questions)

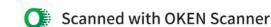
Answer all questions.

Each question carries a weightage 1.

- Find the order of (2,6) in the group Z<sub>4</sub> × Z<sub>12</sub>.
- 2. List all abelian groups up to isomorphism of order 16.
- 3. Let X be a G-set. Prove that  $G_x$  is a subgroup of G for each  $x \in X$ .
- 4. Let  $\phi: \mathbb{Z}_{12} \to \mathbb{Z}_3$  be the homomorphism, where  $\phi(1) = 2$ . Find Ker  $\phi$ .
- 5. Find isomorphic refinements of  $\{0\} < 10 \mathbb{Z} < \mathbb{Z}$  and  $\{0\} < 25 \mathbb{Z} < \mathbb{Z}$ ,
- 6. Find the number of Sylow 3-Subgroups of a group of order 54.
- 7. Give a presentation of  $\mathbb{Z}_4$  involving three generators.
- 8. Find the multiplicative inverse of 1+i+2j+k in the ring of quaternions.

 $(8 \times 1 = 8 \text{ weightage})$ 

Turn over



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# Section B (Paragraph Type Questions)

Answer any two questions from each module

Each question carries a weightage 2.

### MODULE I

- 9. If H is a normal subgroup of a group G, then show that the left coset multiplication is well d
- 10. List all elements of  $\mathbb{Z}_2 \times \mathbb{Z}_4$ . Is this group cyclic. Justify

by the equation (aH)(bH) = abH.

11. Prove that factor group of a cyclic group is cyclic.

Module II

- 12. Find all composition series of  $\mathbb{Z}_{48}$
- 13. Prove that every group of prime power order is solvable
- 14. Prove that every group is a homomorphic image of a free group.

### Module III

- 15. Let  $\phi_{\pi}: \mathbb{Q}[x] + \mathbb{R}$  be the evaluation homomorphism with  $\phi_{\pi}(x) = \pi$ . Find the kernel of
- 16. Factorize the polynomial  $f(x) = 2x^3 + 3x^2$ 7xOT into linear factors in  $\mathbb{Z}_{11}[x]$ .
- 17. Prove that  $\mathbb{Z}/n\mathbb{Z} = \mathbb{Z}_n$ .

 $(6 \times 2 = 12^{W})$ 

## Section C (Essay Type Questions)

Answer any two questions

Each question carries a weightage 5.

- 18, a) Define decomposable group. Prove that the finite indecomposable abelian groups the cyclic groups with order a power of a prime
- <u>b</u> If X is a G-set, then prove that the function  $\sigma_g: X \to X$  defined by  $\sigma_g(x) = gx$  is  $aP^g$ on X . Also show that the map  $\phi\colon G o S_X$  defined by  $\phi(g)=\sigma_g$ is a homomorphis

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- State and prove First Sylow Theorem.
- 5 Prove that no group of order 15 is simple
- 3 Let H and K be normal subgroups of a group G with  $K \le H$ . Show that G/H = G/K/H/K.
- 6) Find the ascending central series of the group  $S_3$
- State and prove Eisenstein Criterion for irreducibility of a polynomial
- Show that the polynomial  $x^5 + 6x^3 + 4x + 10$  is irreducible over  $\mathbb{Q}$ .