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Name....................................

Reg. No....................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 session for SDE/Private students]

(CBCSS)

Mathematics

MTH 1C 05—NUMBER THEORY

(2019 Admission onwards)

{Covid instructions are not applicable for Pvt/SDE students (November 2020 session)}

Time : Three Hours

Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions from this part.*
Each question has weightage 1.

1. Prove that if $2^n + 1$ is prime, then n is a power of 2.

2. If f is a multiplicative function , prove that $f^{-1}(n) = \mu(n) f(n)$ for every square free n.

3. State Selberg's identity.

4. For $x > 0$, show that $0 \le \frac{\psi(x)}{x} - \frac{\vartheta(x)}{x} \le \frac{(\log x)^2}{2\sqrt{x}\log 2}$.

5. For $n \ge 1$, show that $\sum_{n > x} \frac{1}{n^s} = O\left(x^{1-s}\right)$, if $s > 1$.

Turn over

6. Evaluate the Legendre's symbol (7/11).

7. State Euler's criterion for Legendre's symbol.

8. Determine whether -104 is a quadratic residue or non-residue of the prime 997.

($8 \times 1 = 8$ weightage)

Part B

Answer any **six** *questions by choosing* **two** *questions from each unit.*
Each question carries a weightage of 2.

Unit I

9. If $n \geq 1$, show that $\log n = \sum_{d/n} \wedge(d)$.

10. If f is an arithmetical function with $f(1) \neq 0$, show that there is a unique arithmetical fun f^{-1} such that $f * f^{-1} = f^{-1} * f = I$.

11. State and prove Euler's summation formula.

Unit II

12. For every integer $n \geq 2$, show that $\frac{1}{6}\frac{n}{\log n} < \pi(n) = \frac{6n}{\log n}$.

13. For $n \geq 1$, then the n^{th} prime p_n satisfies the inequalities $\frac{1}{6}\log n < p_n < 12\left(n\log n + n\log\frac{12}{e}\right.$

14. State and prove Abel's identity.

Unit III

15. Let p be an odd prime. Then for all n prove that $(n/p) \equiv n^{\left(\frac{p-1}{2}\right)} (\bmod\, p)$.

16. If p and q are distinct odd primes, then show that $(p|q)(q|p) = (-1)^{\frac{(p-1)(q-1)}{4}}$.

17. Describe briefly about RSA cryptosystems.

($6 \times 2 = 12$ weightage)

Part C

Answer any **two** *questions.*
Each question carries weightage of 5.

18. Let f be multiplicative. Then f is completely multiplicative if and only if $f^{-1}(n) = \mu(n) f(n), \forall n \geq 1$.

19. Prove that there is a constant A such that $\sum_{p \leq x} \frac{1}{p} = \log(\log x) + A + O\left(\frac{1}{\log x}\right)$.

20. Show that the Diophantine equation $y^2 = x^3 + k$ has no solutions if k has the form $k = (4n-1)^3 - 4m^2$ where m and n are integers such that no prime $p \equiv -1 \pmod 4$ divides m.

21. Explain Affine enciphering transformations. In the 27-letter alphabet (with blank = 26), use the affine enciphering transformation with n key $a = 13$, $b = 9$ to encipher the message "HELP ME".

(2 × 5 = 10 weightage)