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Name..................................

Reg. No...............................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 session for SDE/Private students]

(CBCSS)

Mathematics

MTH 1C 02—LINEAR ALGEBRA

(2019 Admission onwards)

{Covid instructions are not applicable for PVT/SDE Students (November 2020 sessions)}

Time : Three Hours

Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Let V the vector space of all polynomial functions from a field F into itself. Let T be the linear operator defined as $T(f(x)) = xf(x)$ and D is the differential operator on V. Then prove that $TD \neq DT$.

2. Let 0 be the zero vector of a vector space V. Prove that $v0 = 0$ for all $v \in V$.

3. Let $\mathbb{F}$ be a field and let T be the linear operator on $\mathbb{F}^2$ defined by $(x_1, x_2) = (x_1 = 0)$. Find the matrix of T with respect to the standard ordered basis of $\mathbb{F}^2$.

4. Check whether $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined as $T(x, y) = (x - y, 0)$ is a linear transformation or not.

5. Let V be a vector space over $\mathbb{R}$ of all differentiable functions on $[a, b]$. Let $A : V \to R$ defined as $Af = f'(c)$ where c is a fixed real number in $[a, b]$. Is f a linear functional ? Why ?

Turn over

2

14

6. Find the rank, null space and nullity of identity transformation on a finite dimensional space V.

7. Let $W_1, \ldots, W_2$ be subspaces of a vector space V. Prove that W_1 and W_2 are linearly indepen… and only if $W_1 \cap W_2 = \{0\}$.

8. State Bessel's inequality.

(8 × 1 = 8 weig…

Part B

Answer any **six** *by choosing* **two** *questions from each unit.*
Each question has weightage of 2.

Unit I

9. Prove that $\mathbb{R}^3$ is a vector space over $\mathbb{R}$. Is it a vector space over $\mathbb{C}$?

10. Let V we be a finite dimensional vector space of dimension n. Prove that any subset of V … more than n elements is linearly dependent.

11. Let V be an n-dimensional vector space over the field $\mathbb{F}$ and let W be an m-dimensional … space over $\mathbb{F}$. Prove that the dimension of the space L (V, W) is mn.

Unit II

12. Let A be an $n \times n$ triangular matrix over the field F. Prove that the characteristic values of … its diagonal entries.

13. Let T be a linear operator on a finite dimensional vector space V. Prove that if c is a charact… value of T then det $(T - cI) = 0$.

14. Let W_1 and W_2 be subspaces of a finite dimensional vector space. Prove that $W_1 = W_2$ if an… if $W_1^0 = W_2^0$.

Unit III

15. Prove that an orthogonal set of non-zero vectors is linearly independent.

16. True or false : If E_1, E_2 are projections onto independent subspaces then $E_1 + E_2$ is also a proje… Justify.

17. Let T be a diagonalizable operator having characteristic values 0 and 1 only. Prove that T … projection.

(6 × 2 = 12 weight…

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Let A be an $m \times n$ matrix with entries from a field $\mathbb{F}$. Define row rank of A and also prove that row rank of A is same as column rank of A.

19. Let E_1, E_2 be linear operators on the space V with $E_1 + E_2 = I$. Prove that $E_i^2 = E_i$ for $i = 1, 2$ if and only if $E_1 E_2 = 0$.

20. Let θ be a real number. Prove that the following two matrices are similar over $\mathbb{C}$,

$$A = \begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix}, B = \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}.$$

21. Let T be a linear operator on a finite dimensional vector space V with $c_1, ..., c_k$ be the distinct characteristic values of T and let W_i be the space of characteristic values associated with characteristic value ci. Prove that if $W = W_1 + ... + W_k$ then $\sum_{i=1}^{k} \dim W_i = \dim W$.

(2 × 5 = 10 weightage)