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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2021

[November 2020 session for SDE/Private Students]

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admission onwards)

{Covid instructions are not applicable for PVT/SDE students (November 2020 session)}

Time : Three Hours Maximum : 30 Weightage

General Instructions

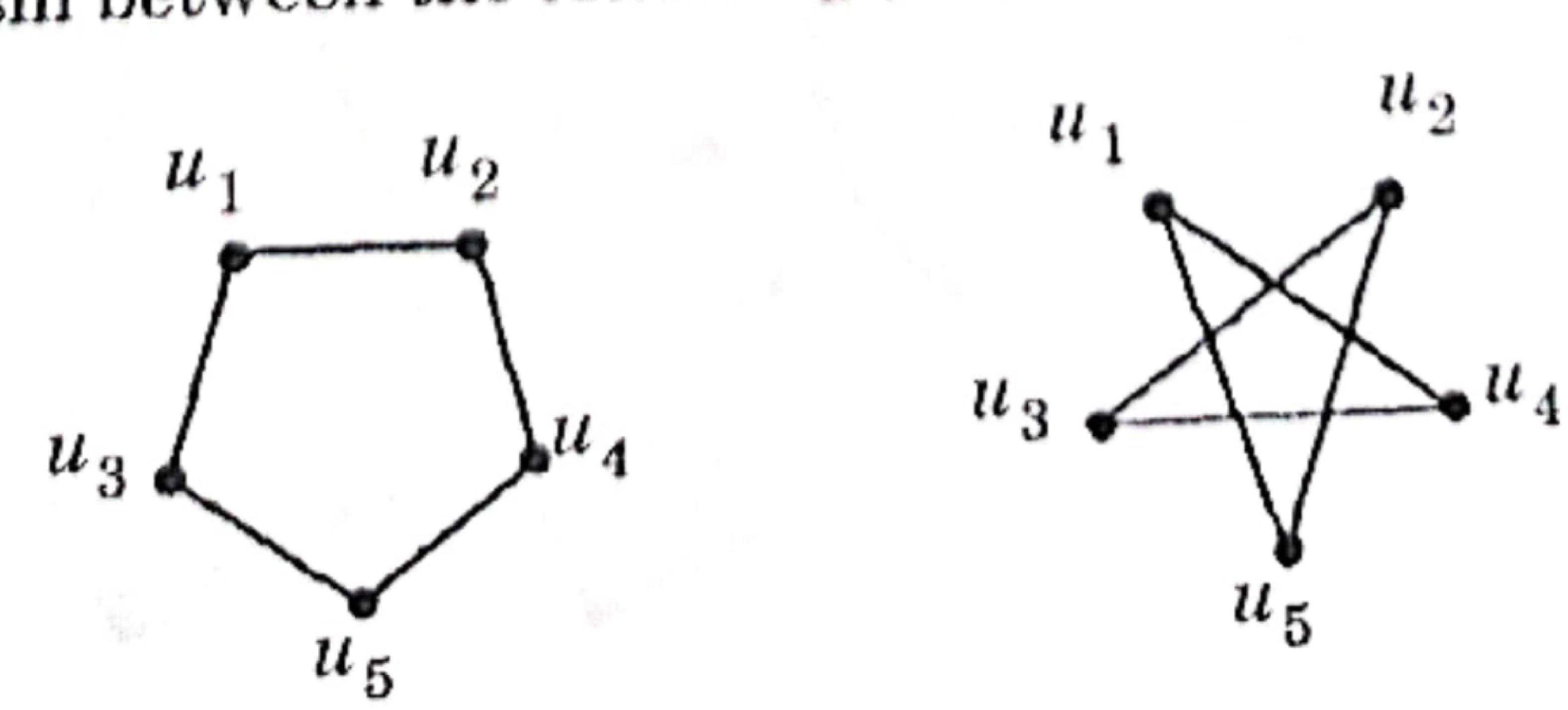
1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *The instruction if any, to attend a minimum number of questions from each sub section / sub part / sub division may be ignored.*
4. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A

Answer **all** *questions.*

Each question carries weightage 1.

1. Define partial ordering and give an example.
2. What is a chain ? Give an example.
3. Find the maximal elements of the poset X = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} under the partial order $a \mid b$ if a divides b.
4. Define an isomorphism between the following graphs :



Turn over

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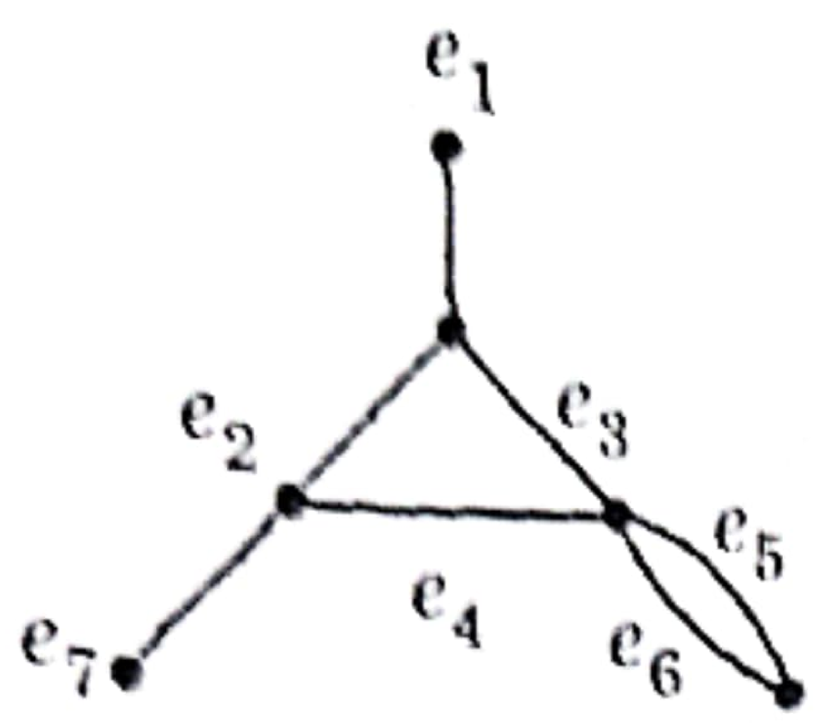
5. Draw the line graph of the following graph



6. If G is a simple planar graph with at least 3 vertices, prove that $m \leq 3n - 6$, where m and

number of edges and vertices of G, respectively.

7. Let $w = a_1 a_2 \ldots a_n$ be a string. Find length of w and reverse of w.

8. Define star-closure of a language.

($8 \times 1 =$

Part B

Answer any **six** *questions by choosing* **two** *questions from each unit.*
Each question carries a weightage of 2.

Unit I

9. Let $(X, \leq)$ be a poset and A a non-empty finite subset of X. Prove that A has at least

element.

10. Let X be a set and let $\leq$ be a binary relation defined on X which is reflexive and transitive

a binary relation on X by xRy if $x \leq y$ and $y \leq x$. Prove that R is an equivalence relation

11. Let m be the largest possible number of mutually incomparable elements in a poset X

X cannot be expressed as a union of less than m chains.

Unit II

12. In any graph G, prove that the number of vertices of odd degree is even.

13. Prove that a vertex v of a connected graph G with at least three vertices is a cut vertex of G

only if there exist vertices u and w of G distinct from v such that v is in every $u-w$ path

14. Prove that a simple graph is a tree if and only if every pair of vertices is connected by a

path.

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Unit III

15. Find a grammar that generates the language $L = \{a^n b^{n+1} : n \geq 0\}$.

16. Distinguish between deterministic automata and non-deterministic automata.

17. Let $M = (Q, \Sigma, \delta, q_0, F)$ be a deterministic finite accepter and let G_M be its associate transition graph. Then prove that for every $q_i, q_j \in Q$ and $w \in \Sigma^*$, $\delta^*(q_i, w) = q_j$ if and only if there is in G_M a walk with label w from q_i to q_j.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question carries a weightage of 5.

18. Let A be a chain in a poset X and $|A|$ denotes the length of A. If $X = P(B)$ where B is a set with n elements and the partial order on X is by set inclusion, prove that

    (i) For S, T $\in$ X, T covers S iff $S \subset T$ and $T - S$ is singleton set.

    (ii) The longest chain in X is of length $n + 1$.

    (iii) The number of such chains is n !.

19. (i) Prove that a connected graph G with at least two vertices contains at least two vertices that are not cut vertices.

    (ii) Prove that a simple cubic connected graph G has a cut vertex if and only if it has a cut edge.

20. (i) A graph G is planar if and only if each of its blocks is planar.

    (ii) Define dual of a planar graph. Draw a planar representation of K_4 and its dual.

21. Find a d.f.a. that :

    (i) Recognizes the set of all strings on $\Sigma = \{a, b\}$ starting with prefix ab.

    (ii) Accepts all strings on $\{0,1\}$ except those containing the substring 001.

(2 × 5 = 10 weightage)