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Name..

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FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 03—REAL ANALYSIS I

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

General Instructions

1. *In cases where choices are provided, students can attend* **all** *questions in each section.*
2. *The minimum number of questions to be attended from the Section / Part shall remain the same.*
3. *There will be an overall ceiling for each Section / Part that is equivalent to the maximum weightage of the Section / Part.*

Part A (Short Answer Questions)

Answer **all** *the questions.*
Each question carries 1 *weightage.*

1. Construct a bounded set of real numbers with exactly three limit points.
2. Let Y be an open subset of a metric space. If a subset E of Y is open relative to Y, then prove that E is open in X.
3. Let f be a continuous mapping of a metric space X into a metric space Y. If $E \subset X$, then prove that $f(\bar{E}) \subset \overline{f(E)}$.
4. Give an example of a differentiable function f on $\mathbb{R}$ such that f' is not continuous at 0.
5. Let f_1, f_2 be bounded functions and α be a monotonic increasing function on $[a, b]$. If f_1 and f_2 are Riemann-Stieltjes integrable with respect to α on $[a, b]$, then prove that $f_1 + f_2$ is Riemann-Stieltjes integrable with respect to α on $[a, b]$.
6. Let f be a bounded function and α be a monotonic increasing function on $[a, b]$ such that $|f|$ is Riemann-Stieltjes integrable with respect to α. Is f Riemann-Stieltjes integrable with respect to α? Justify your answer.

Turn over

7. Let γ be a curve in the complex plane, defined on $[0,2\pi]$ by $\gamma(t)=e^{2\pi i t \sin\frac{1}{t}}$. Prove that γ is no rectifiable.

8. If the sequences $\{f_n\}$ and $\{g_n\}$ converge uniformly on a set E, then prove that then sequen $\{f_n+g_n\}$ converge uniformly on E.

(8 × 1 = 8 weightag

Part B

Answer any **two** *questions of each unit.*
Each question has weightage 2.

Unit I

9. Let A be the set of all sequences whose elements are the digits 0 and 1. Prove that A is countab

10. Prove that a closed subset of a compact space is compact.

11. Let X be a connected metric space, Y be a metric space and let $f: X \to Y$ be a surjective continuo map. Prove that Y is connected.

Unit II

12. Let f be a real function defined on $[a, b]$ and let f be differentiable on (a, b). If $f'(x) \geq 0$ for $x \in (a,b)$ then prove that f is monotonically increasing.

13. If f is differentiable on $[a, b]$, then prove that f' cannot have any simple discontinuities on $[a,$

14. If f is Riemann-Stieltjes integrable with respect to α on $[a, b]$ and if $a < c < b$, then prove th f is Riemann-Stieltjes integrable with respect α on $[a, c]$ and on $[c, b)$ and

$$\int_a^c f\,d\alpha + \int_c^b f\,d\alpha = \int_a^b f\,d\alpha.$$

Unit III

15. Prove that the series $\sum_{n=1}^{\infty} (-1)^n \frac{x^2+n}{n^2}$ converges uniformly in every bounded interval.

16. Let $\{f_n\}$ be a sequence of integrable functions and let f be an integrable function such that $f_n \to$ Is it true that $\int f\,dx = \lim \int f_n\,dx$? Justify your answer.

17. Let $\mathcal{C}$ (X) denote the set of all complex valued, continuous, bounded functions defined on a met space X. Prove that $\mathcal{C}$ (X) is a complete metric space with respect to the metric :

$d(f,g) = \sup_{x \in X} |f(x) - g(x)|$.

(6 × 2 = 12 weightag

Part C

Answer any **two** *from the following four questions* (18–21).
Each question has weightage 5.

18. (a) Prove that a subset E of a metric space is open if and only if its complement E^c is closed.

(b) Prove that monotonic functions have no discontinuities of the second kind.

19. (a) Let f be a bounded function and a be a monotonic increasing function on $[a, b]$. If P_1 is a refinement of P, then prove that :

$$L(P,f,\alpha) \leq L(P_1,f,\alpha)$$

(b) Let f be a bounded function and α be a monotonic increasing function on $[a, b]$. If f is continuous on $[a, b]$, then prove that f is Riemann-Stieltjes integrable with respect to α on $[a, b]$.

20. (a) Prove that every uniformly convergent sequence of bounded functions is uniformly bounded.

(b) Let $\{f_n\}$ be a sequence of functions, differentiable on $[a, b]$ and such that $\{f_n(x_0)\}$ converges for some point x_0 on $[a, b]$. If $\{f_n'\}$ converges uniformly on $[a, b]$, then prove that $\{f_n\}$ converges uniformly on $[a, b]$, to a function f, and

$$f'(x) = \lim_{n \to \infty} f_n'(x) \text{ for all } x \in [a,b].$$

21. (a) Prove that there exists a real continuous function on the real line which is nowhere differentiable.

(b) Let K be a compact metric space and let $f_n \in C(K)$ for $n = 1, 2, 3, \ldots$ and $\{f_n\}$ converges uniformly on K. Prove that $\{f_n\}$ is equicontinuous on K.

(2 × 5 = 10 weightage)