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D 93426 (Pages : 4) Name..............................

Reg. No..............................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 04—DISCRETE MATHEMATICS

(2019 Admissions)

Time : Three Hours Maximum : 30 Weightage

Part A

Answer **all** *questions in Part A.*
Each question carries a weightage of 1.

1. State a necessary and sufficient condition for two simple graphs G and H to be isomorphic.
2. Prove or disprove : No loop can belong to an edge cut.
3. Is it true that every eulerian graph is connected. Justify.
4. If G is a connected plane graph, show that a cut edge of G belong to exactly one face.
5. Define maximal element and maximum element. Illustrate with an example.
6. If $(X, +, ., ')$ is a Boolean algebra, prove that : (i) $x+x=x$ and (ii) $x \cdot x = x$ for $x \in X$.
7. Prove that $(uv)^R = v^R u^R$ for all $u, v \in \sum^+$.
8. Define a finite automaton. Illustrate with an example.

(8 × 1 = 8 weightage)

Part B

Answer **six** *questions in Part B choosing* **two** *from each unit.*
Each question carries a weightage of 2.

Unit I

9. (a) Show that the number of edges of a simple graph with n vertices and w components is :

$$\frac{(n-w)(n-w+1)}{2}.$$

(b) Is it true ; if G is a simple graph with $\delta \geq \frac{n-2}{2}$, then G is connected. Justify.

Turn over

10. (a) Prove that a simple graph is a tree if and only if any two distinct vertices are connected by a unique path.

(b) Show that the number of edges of a tree on a vertices is $n - 1$.

11. (a) If G is a plane graph and f is a face of G then prove that there exists a plane embedding of G in which f is the exterior face.

(b) Prove or disprove : All planar embedding of a given planar graph have the same number of faces.

Unit II

12. (a) Let X be a set and $\leq$ be a binary relation on X which is reflexive and transitive. Define R on X by x Ry if $x \leq y$ and $y \leq x$. Verify whether R is an equivalence relation.

(b) Prove or disprove : A longest, chain in a partially ordered set $(X \leq)$ is maximal, but the converse need not hold.

13. If $(X, +, ., ')$ is a Boolean algebra,, prow that, for all $x, y, z \in X$.

(a) $x + (y + z) = (x + Y) + z$ and (b) $(x')' = x$.

14. (a) What is the disjunctive normal form of a Boolean function.

(b) Write the Boolean function $f(x_1, x_2, x_3) = (x_1 + x_2')x_3' + x_2x_1'(x_2 + x_1'x_3)$ in its disjunctive normal form.

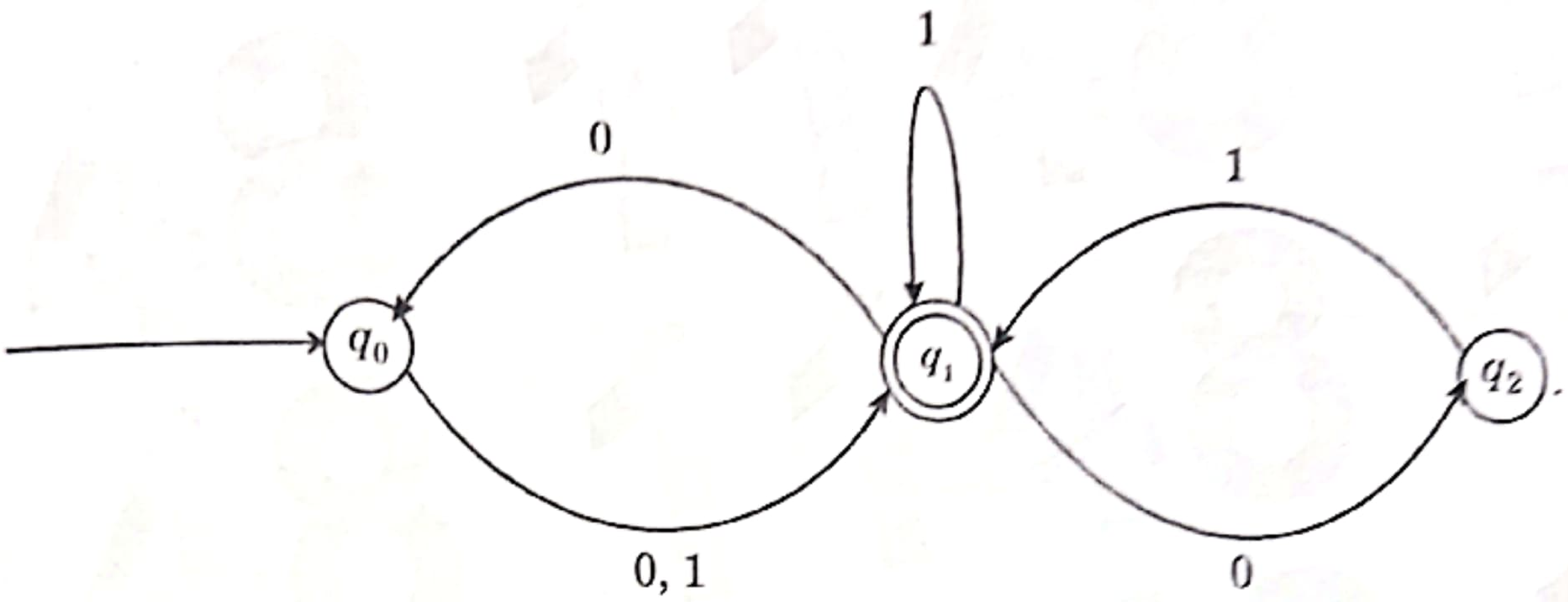
Unit III

15. (a) Find dfa that accepts the set of all strings with exactly one a where $\sum = \{a, b\}$.

(b) Find an nfa that accepts $\{a\}^*$ and is such that if in its transition graph a single edge is removed without any other changes, the resulting automaton accepts $\{a\}$.

16. Find a dfa that accepts the set of all strings on $\sum = \{a, b\}$ staring with the Prefix ab '

17. Which of the given strings are accepted by the following automaton : (i) 01001 ; (ii) 10010 ; (iii) 000.



(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions in Part C.*

Each question carries a weightage of 5.

18. (a) Let T be a graph with n vertices, then prove the following are equivalent :

    (i) T is a tree.

    (ii) T has no cycles and has $n - 1$ edges.

    (iii) There exists a unique path between any two vertices in T.

    (b) Find all non-isomorphic trees with 5 vertices.

19. (a) Prove that K_5 is non-planar.

    (b) For any simple planar graph G, prove that $\delta(G) \leq 5$.

20. (a) Show that every Boolean function on n variables $x_1, x_2, \ldots, x_n$ can be uniquely expressed as the sum of terms of the form $x_1^{\epsilon_1} x_2^{\epsilon_2} \ldots x_n^{\epsilon_n}$ where each $x_i^{\epsilon_i}$ is either x_i or x_i'.

    (b) Show that every finite Boolean algebra is isomorphic to a power set Boolean algebra.

Turn over

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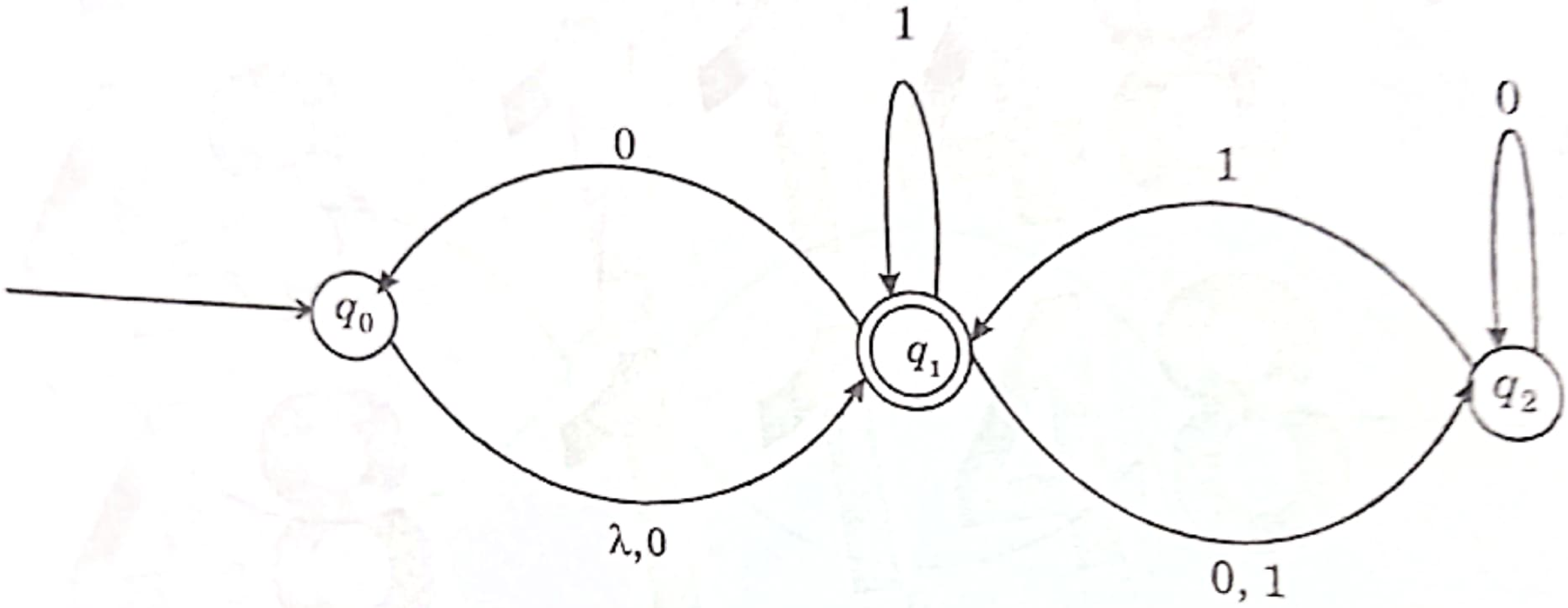
21. Convert the *nfa* given by the following diagram into an equivalent *dfa*.



(2 × 5 = 10 weightage)