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Name...

Reg. No....................................

FIRST SEMESTER M.Sc. DEGREE (REGULAR/SUPPLEMENTARY) EXAMINATION, NOVEMBER 2020

(CBCSS)

Mathematics

MTH 1C 01—ALGEBRA I

(2019 Admissions)

Time : Three Hours

Maximum : 30 Weightage

Part A

Answer **all** *questions.*
Each question has weightage 1.

1. Verify whether $\phi(x,y)=(x+1,y)$ is an isometry of the plane.

2. Find a generator of the cyclic group $\mathbb{Z}_2 \times \mathbb{Z}_3$.

3. Describe all abelian groups of order 20 upto isomorphism.

4. Find all subgroups of the quotient group $\mathbb{Z}/6\mathbb{Z}$.

5. Let G be a group of order 18. Find the number of 3-Sylow subgroups of G.

6. Give all elements of the group given by the presentation $\left(a : a^5 = 1\right)$.

7. Find the inverse of $2i + j + k$ in the ring of quaternions.

8. Let $\phi : \mathbb{Z}\times \to \mathbb{Z}\times\mathbb{Z}$ be the homomorphism defined by $(x,y)\mapsto(x,0)$. Find $\ker\phi$.

(8 × 1 = 8 weightage)

Part B

Answer any **six** *questions, choosing two from each unit.*
Earth question has weightage 2.

Unit 1

9. Verify whether $\mathbb{Z}_5 \times \mathbb{Z}_6$ is a cyclic group.

10. Show that $\mathbb{Z}/5\mathbb{Z}$ is isomorphic to $\mathbb{Z}_5$.

11. Verify whether $\mathbb{Z}_6$ is simple.

Turn over

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Unit 2

12. Show that (1 2 3) and (1 3 2) are conjugates in the symmetric group S_3.

13. Verify whether the following series of groups are isomorphic :

$0 < (5) < \mathbb{Z}_{15}$ and $0 < (3) < \mathbb{Z}_{15}$.

14. Show that every group of order 15 is cyclic.

Unit 3

15. Show that the ring of all endomorphisms of the group $\mathbb{Z}$ of integers is commutative.

16. Let $\phi_2 : \mathbb{Q}[x] \to \mathbb{Q}$ be the evaluation homomorphism with $\phi_2(x) = 2$. Find the kernal of ϕ_2.

17. Verify whether $x^3 + 3x + 2 \in \mathbb{Z}_5[x]$ is irreducible.

(6 × 2 = 12 weightage)

Part C

Answer any **two** *questions.*
Each question has weightage 5.

18. (a) Let G be a group and H be a normal subgroup of G.

    i. Show that the coset multiplication $(a\text{H})(b\text{H}) = (ab)\text{H}$ is well defined.

    ii. Verify that G / H is a group.

    (b) Describe the factor group $\mathbb{Z}_{20}/\text{H}$ where H is the subgroup generated by 5.

19. (a) Let H, K be groups and G = H × K. Show that :

    i. $\bar{\text{H}} = \{(h, e) : h \in \text{H}\}$ is a normal subgroup of G where is the identity of K.

    ii. $\text{G}/\bar{\text{H}}$ is isomorphic to K.

    (b) Show that factor groups of cyclic groups are cyclic.

20. Let G be a group and N be a normal subgroup of G and H be any subgroup of G. Show that :

    (a) H N = N H.

    (b) H N is a subgroup of G.

    (c) $\text{H} \cap \text{N}$ is a normal subgroup of G.

    (d) H N / N is isomorphic to $\text{H}/(\text{H} \cap \text{N})$.

21. (a) Let F be a field and $f(x) \in \text{F}[x]$. Show that $a \in \text{F}$ is a zero of $f(x)$ if and only if $(x - a)$ is a factor of $f(x)$.

    (b) Prove that a polynomial of degree 3 in F [x] is irreducible if and only if it has no zero in F.

(2 × 5 = 10 weightage)