C 83627 **(Pages : 3)** Name

Reg. No..................

SECOND SEMESTER M.Sc. DEGREE EXAMINATION, JUNE 2015

(CUCSS)

Mathematics

MT 2C 10—NUMBER THEORY

Time : Three Hours **Maximum : 36 Weightage**

Part A

Answer **all** *questions.*
Each question carries a weightage of 1.

1. Find all integers x such that $\varphi(n) = \varphi(2n)$.
2. Show that 9 $(mn) = \varphi(m)\varphi(n)$ if (m, n) =1.
3. Define completely multiplicative function.
4. Define divisor functions $\sigma_\alpha(n)$ for n 1 and show that they are multiplicative.
5. If f and g are arithmetical functions, then show that $(f * g)' = f' * g + f * g'$.
6. Show that if $a > 0$ and $b > 0$, then $\lim \frac{(ax)}{\pi(bx)} = \frac{a}{b}$
7. Let (a, m) = 1. Show that the linear congruence $ax \equiv b$ (mod m) has exactly one solution.
8. Determine the quadratic residues and non-residues modulo 11.
9. Determine those odd primes p for which 3 is a quadratic residue.
10. Show that if p is an odd positive integer then $(2/p) = (-1)^{\frac{p^2-1}{8}}$.
11. Prove that the product of two linear enciphering transformations is also a linear enciphering transformation.
12. Write a short note on enciphering key.
13. What is classical cryptosystem ?
14. State the map coloring problem and translate it to a graph coloring problem.

(14 x 1 = 14 weightage)

Turn over

Part B

Answer **any seven** *questions.*
Each question carries a weightage of **2.**

15. **Show that for n 1,** $\varphi(n) = n \prod_{p/n} \left(1 - \frac{1}{p}\right)$.

16. **Let** f **be a multiplicative function. Show that** f **is completely multiplicative iff** $f^{-1}(n) = \mu(n) f(n)$ **for all n 1.**

17. **State and prove Euler's summation formula.**

18. **Show that for** $x \geq 2$**;** $\sum_{p \leq x} \left[\frac{x}{p}\right] \log p = x \log x - x + O(\log x)$.

19. **Show that for any prime** p **5;** $\sum_{k=1}^{1 (- 1)} = 0 \pmod{p^2}$.

20. **Let** p **be an odd prime. Show that for all n ;** $(n/ \quad \left(\frac{p-1}{2}\right) \pmod{p}$.

21. **Show that given any integer** $k > 0$ **there exists a lattice point (a,** b**) such that none of the lattice points (a** $+ r$, $b + s$), $0 < r \leq k$, $0 < s \leq k$ **is visible from the origin.**

22. **Find the inverse of A** $= \begin{pmatrix} n & 3 \\ 7 & 8 \end{pmatrix} \in M_2\left(\frac{Z}{26Z}\right)$.

23. **Solve the following system of simultaneous congruences :**

    $17x + 11y \equiv 7 \pmod{29}$
    $13x + 10y \equiv 8 \pmod{29}$.

24. **Write a note on the ElGamal cryptosystem.**

(7 x 2 = 14 weightage)

Part C

Answer any two questions.
Each question carries a weightage of **4.**

25. **Show that the set of all arithmetical functions f with $f(1) \neq 0$ forms an abelian group with respect to the Dirichlet product.**

26. **Let p_n deonte the nth prime. Prove that the following are equivalent :**

    (i) $\lim_{x \to \infty} \frac{\pi(x) \log x}{x} = 1.$

    (ii) $\lim_{x \to \infty} \frac{\pi(x) \log \pi(x)}{x} = 1.$

    (iii) $\lim_{n \to \infty} \frac{p_n}{n \log n} = 1.$

27. **State and prove Quadratic reciprocity law.**

28. **Let $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in M_2\left(\mathbb{Z}/N\mathbb{Z}\right)$ and set D = a. Prove that the following are equivalent :**

    (a) **g c d = (D, N) =1.**

    (b) **A has an inverse.**

    (c) **If x and y are not both 0 in $\frac{\mathbb{Z}}{N\mathbb{Z}}$, then $A\begin{pmatrix} x \\ y \end{pmatrix} \neq \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.**

    (d) **A gives a one to one correspondence of $\left(\frac{\mathbb{Z}}{N\mathbb{Z}}\right)^2$ with itself.**

(2 x 4 = 8 weightage)