D 90231 (Pages : 3) Name.................................

Reg. No.................................

FIFTH SEMESTER B.Sc. DEGREE EXAMINATION, NOVEMBER 2020

(CUCBCSS—UG)

Mathematics

MAT 5B 06—ABSTRACT ALGEBRA

Time : Three Hours Maximum : 120 Marks

Section A

Answer **all** *questions.*

Each question carries 1 *mark.*

1. Define a Group.
2. Fill in the blanks : The units in the ring of integers $\mathbb{Z}$ are ———.
3. Write the order of the permutation (1, 2) (1 9 8) in S_9.
4. Give an example of a finite group of order 4 which is not cyclic.
5. Calculate the remainder obtained when 45^{72} is divided by 73.
6. Find the inverse of the product (7, 5) (2, 5, 7) in S_7. Is the inverse a cycle ?
7. What is the characteristic of the ring $< \mathbb{Z}_9 , +_9, \times_9 >$.
8. Give an example for an integral domain which is not a field.
9. What is the necessary condition for a homomorphism ϕ from a group G to G' to be injective.
10. Define normal subgroup of a group.
11. What is the index of A_n in S_n.
12. Define a cyclic group and give an example.

(12 × 1 = 12 marks)

Section B

Answer at least **eight** *questions.*

Each question carries 6 *marks.*

All questions can be attended.

Overall Ceiling 48.

13. Write criteria to be checked to determine whether a function $\phi : S \to S'$ is an isomorphism of a binary structure $< S, * >$ with $< S', *' >$.

Turn over

14. Is $\mathbb{Z}^+$ a group under usual addition ? Establish your claim.

15. Solve : $x^2 - 1 = 0$ in the field $\mathbb{Z}_p$.

16. Show that every field is an integral domain.

17. Find the multiplicative inverse of 53 in $\mathbb{Z}_{57}$.

18. Construct group table for the Klein group. What is the order of every element in this group ?

19. Define kernel of a group homomorphism. Find the $\ker(\phi)$ for $\phi : \mathbb{R} \to \mathbb{R}$ defined by $\phi(x) = 0$ for all $x \in \mathbb{R}$.

20. Define a ring. Give an example of a non-commutative ring.

21. Define center of a group and show that center of the symmetric group S_3 is the trivial group.

22. In any ring R, show that $a.0 = 0 = 0.a$ and $a.(-b) = -(a.b)$ for all a, b in R.

23. Show that for any group, its identity element and inverse of any element are unique.

24. Evaluate the product of (2, 3) and (3, 5) in $\mathbb{Z}_5 \times \mathbb{Z}_9$.

25. Show that $a^2 - b^2 = (a-b)(a+b)$ in a ring R if and only if R is commutative.

26. Define factor group and give an example.

(8 × 6 = 48 marks)

Section C

Answer at least **five** *questions.*
Each question carries 9 *marks.*
All questions can be attended.
Overall Ceiling 45.

27. Show that the binary structure $<\mathbb{R},+>$ with operation the usual addition is isomorphic to the structure $<\mathbb{R}^+,.>$ where . is the usual multiplication.

28. (a) State and prove Lagrange's theorem ; and (b) Establish one of its corollary.

29. Show that the subset S of $M_n(\mathbb{R})$ consisting of all invertible $n \times n$ matrices under matrix multiplication is a group.

30. Show that every permutation σ of a finite set is a product of disjoint cycles.

31. Let G and G' be groups and let $\phi : G \to G'$ be one to one function such that $\phi(xy) = \phi(x)\Phi(y)$ for all $x, y \in G$. Then prove that $\phi[G]$ is a subgroup of G' and ϕ provides an isomorphism of G with $\Phi[G]$.

32. Show that subgroup a cyclic group is cyclic.

33. Show that M is a maximal normal subgroup of G if and only if G/M is simple.

34. Show that the cancellation law in a ring R holds if and only if it has no zero divisors.

35. Find all solutions of the congruence $12x \equiv 27(\mathrm{mod}\,18)$.

(5 × 9 = 45 marks)

Section D

Answer any **one** *question.*
The question carries 15 *marks.*

36. (a) Show that $|< a^s >| = |< a^t >|$ if and only if g.c.d (n, s) = g.c.d (n, t) where $n = |< a >|$.

(b) Show that the only subgroups of $\mathbb{Z}$ are the form $n\mathbb{Z}$ for $n \in \mathbb{Z}$.

37. State Cayley's theorem and give the proof in detail.

38. (a) Show that any two fields of quotients of an integral domain are isomorphic.

(b) Prove or disprove : Factor group of a cyclic group is cyclic.

(1 × 15 = 15 marks)