C 80916

Maths

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Name..............................

Reg. No..............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION, APRIL 2020

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

Time : Three Hours

Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *the* **twelve** *questions.*

Each question carries 1 *mark.*

1. If α, β, γ are the roots of $2x^3 + 3x^2 - x - 1 = 0$. Find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.
2. State the Fundamental theorem of algebra.
3. Find a cubic equation, two of whose roots are given by 1, 3 + 2*i*.
4. What do you mean by reciprocal equation. Give example.
5. What is the rank of the identity matrix of order *n*.
6. If $A = [a_{i,j}]$ is an m × n matrix and $a_{i,j} = 7$ for all i, j then rank of A is ————.
7. A system of m homogeneous linear equations AX = 0 in n unknowns has only trivial solution if ————.
8. For what values of a the system of equations $ax + y = 1, x + 2y = 3, 2x + 3y = 5$ are consistent.
9. If the number of variables in a non homogeneous system AX = B is n then the system possesses a unique solution if ————.
10. Find the parametric equation of a line through P(3,-4,-l) and parallel to the vector $i + j + k$.
11. Find the unit tangent vector of the helix $r(t) = \cos t i + \sin t j + tk$.
12. Write equations relating rectangular and cylindrical coordinates.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*

Each question carries 2 *marks.*

13. Solve $4x^3 - 24x^2 + 23x + 18 = 0$. Given that the roots are in arithmatic progression.
14. Transform $x^3 - 6x^2 + 5x + 12 = 0$ into an equation lacking second term.

Turn over

15. If α, β, γ are the roots of $ax^3 + 3bx^2 + 3cx + d = 0$. Find the value of $(\alpha^2+1)(\beta^2+1)(\gamma^2+1)$.

16. If $A = \begin{pmatrix} -2 & -1 \\ 5 & 4 \end{pmatrix}$ Find A^{-1}.

17. Prove that the characteristic roots of Hermitian matrix are real.

18. If α is an eigen value of a non singular matrix A, prove that $\frac{|A|}{\alpha}$ is an eigen value of adj A.

19. Show that the product of characteristic roots of a square matrix of order n is equal to the determinant of the matrix.

20. Find the value of a for which $r(A) = 3$ where $A = \begin{pmatrix} 2 & 4 & 4 \\ 3 & 1 & 2 \\ 1 & 0 & a \end{pmatrix}$

21. Find the velocity and acceleration vectors of $r(t) = (3\cos t)i + (3\sin t)j + t^2 k$.

22. Find an equation for the circular cylinder $4x^2 + 4y^2 = 9$ in cylindrical coordinates.

23. Evaluate $\int_0^1 \left(t^3 i + 7j + (t+1)k\right) dt$.

24. Find the unit tangent vector for the circular motion $r(t) = (\cos 2t)i + (\sin 2t)j$.

(9 × 2 = 18 marks)

Part C (Short Essay)

Answer any **six** *questions.*
Each question carries 5 marks.

25. If α, β, γ are the roots of $x^3 - x - 1 = 0$. Find the equation whose roots are $\frac{1+\alpha}{1-\alpha}, \frac{1+\beta}{1-\beta}, \frac{1+\gamma}{1-\gamma}$.

26. Solve the equation $x^2 - 12x - 65 = 0$ by cardan's method.

27. Solve $x^3 + 6x^2 + 3x + 18 = 0$.

28. Prove that the rank of a non singular matrix is equal to the rank of its reciprocal matrix.

29. Find the rank of $\begin{pmatrix} 4 & -2 & 3 \\ 2 & 0 & 1 \\ 3 & -1 & 2 \\ 1 & -1 & 1 \end{pmatrix}$.

30. Using matrix method solve,

$$x + 2y + z = 2$$
$$3x + y - 2z = 1$$
$$4x - 3y - z = 3$$
$$2x + 4y + 2z = 4$$

31. Find the point in which the line $x = 1 - t, y = 3t, z = 1 + t$ intersects the plane $2x - y + 3z = 6$.

32. Find the distance from the point S (0, 0, 1, 2) to the line $x = 4t, y = -2t, z = 2t$.

33. Find the eigen values and eigen vectors of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.

(6 × 5 = 30 marks)

Part D

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve the equation $6x^5 - 41x^4 + 97x^3 - 97x^2 + 41x - 6 = 0$.

35. Verify Cayley Hamilton theorem for the matrix $A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{pmatrix}$.

36. Find the binormal vector and torsion for the space curve $r(t) = (3\sin t)i + (3\cos t)j + 4tk$.

(2 × 10 = 20 marks)