D 73287 (Pages : 2) Name..................................

Reg. No..................................

FIRST SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2019

(CBCSS—UG)

Mathematics

MTS 1B 01—BASIC LOGIC AND NUMBER THEORY

(2019 Admissions)

Time : Two Hours and a Half Maximum : 80 Marks

Section A

Answer any number of questions.
Each question carries 2 marks.
Maximum 25 marks.

1. What is a *compound proposition* ? Give two simple propositions and combine it to make compound.
2. If $a = 3, b = 5, c = 6$, explain and evaluate the boolean expression $\left[-(a > b)\right] \wedge (b < c)$.
3. What is meant by a *biconditional statement* ? Give an example.
4. Suppose that in a party, there are n guests. Each person shakes hands with everybody else exactly once. Define recursively the number of handshakes $h(n)$ made.
5. Find the sum of $(100)_2$ and $(110)_2$.
6. Define Fibonacci numbers and Lucas numbers.
7. Evaluate (18, 30, 60, 75, 132).
8. Prove that two positive integers a and b are relatively prime if and only if $[a,b] = ab$.
9. If $a \equiv b$ and $b \equiv c$ modulo n, can we say that $a \equiv c$ modulo n ? Justify.
10. Is {2, 4, 6, 8, 10} a complete set of residues modulo 5 ? Why ?
11. When will we say that a is invertible modulo m ? Give an example for an a invertible modulo 8.
12. What is a pseudoprime ? Give an example and verify it.
13. Let m be a positive integer and a any integer with $(a,m) = 1$. Prove that $a^{\phi(m)} \equiv 1 (\text{mod}\, m)$.
14. What is the value of $\phi\left(p^e\right)$ where p is a prime ? Use it to compute $\phi(81)$ and $\phi(625)$.
15. State the fundamental theorem for multiplicative functions and the Gauss theorem for natural number n on sum of $\phi(d)$ where $d|n$.

Turn over

Section B

Answer any number of questions.
Each question carries 5 marks.
Maximum 35 marks.

16. Give a nonconstructive proof to show that there is a prime number > 3.
17. Prove that any postage of Rupees $n \geq 2$ can be made with two and three rupees stamps.
18. Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.
19. State and prove the Duncan's identity.
20. Find the general solution to the LDE $63x - 23y = -7$.
21. Prove that $a \equiv b \pmod{m}$ if and only if a and b leave the same remainder when divided by m.
22. Let p be a prime and a any integer such that $p|a$. Prove that the least residues of the integers a, $2a, 3a, \ldots, (p-1)a$ modulo p are a permutation of the integers $1, 2, 3, \ldots, (p-1)$.
23. Prove that the tau, sigma functions are multiplicative.

Section C

Answer any **two** *questions.*
Each question carries 10 marks.
Maximum 20 marks.

24. (a) Verify that $p \to q \equiv \sim q \to \sim p$ constructing the truth table.

    (b) Simplify the boolean expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ using the laws of logic.

25. (a) If $(a,b) = d$, prove that $(a/d, b/d) = 1$.

    (b) Prove that the gcd of the positive integers a and b is a linear combination of a and b.

26. What is the necessary and sufficient condition for the LDE $ax + by = c$ to be solvable ? State it and prove it.

27. (a) If p is a prime, prove that $(p-1)! \equiv -1 \pmod{p}$.

    (b) Let p be a prime and n any positive integer. Prove that $\frac{(np)!}{n!\,p^n} \equiv (-1)^n \pmod{p}$.